Enrichment: topics you could meet in a Mathematics degree

This resource is one of several available on the FMSP website: www.furthermaths.org.uk/maths-preparation

Algorithms

An **algorithm** is a precise set of instructions which is used to accomplish a specific process. We come across algorithms in every-day life, for example:

- Knitting patterns
- Assembly instructions for flat-pack furniture or a new electronic device
- Cooking recipes

These are often explained in words, sometimes using abbreviated codes to represent words that are used very often. For example, when knitting glossaries such as **http://www.knittinghelp.com/videos/knitting-glossary** describe types of stitches and processes, like k = knit, p = purl and CO = cast on.

Algorithms are often used in mathematics to abbreviate lengthy procedures which would be complicated to explain in words. Methods used include flow charts and pseudocode. (Pseudocode is outlined in a separate document and is commonly used in computing as a 'halfway' between written instructions and a computer programme).

Some examples of algorithms are outlined below.

Zeller's Algorithm

Zeller's algorithm allows you to work out the day of the week corresponding to any date. So, for example, you could work out the day of the week on which you were born. The algorithm is:

1. Let day number = D, month number = M, year = Y
2. If M=1 or M=2, add 12 to M and subtract 1 from Y. Otherwise, do nothing.
3. Let C be the first two digits of Y and K be the last two digits of Y.
4. Add together the integer parts* of (2.6M – 5.39), (K÷4) and (C÷4). Add D and K to this total. Subtract 2C from your answer.
5. Find the remainder when this number is divided by 7.
6. If the remainder is 0 the day is Sunday, if the remainder is 1 the day is Monday, 2 is Tuesday, etc…

* 'integer part' means ignore anything after the decimal point and just use the whole number part.

Task 1

Work out the day of the week on which you were born, or for some other significant date.

You can check your calculations **here**.

The Russian Peasant Algorithm

This algorithm is used to multiply two numbers together without knowing a formal written method such as long multiplication. It is thought the name originates from the publication of the method in a Russian book, outlining the method that peasants used to make calculations. However, there is also evidence that the method has Egyptian roots.

The steps of the algorithm are:

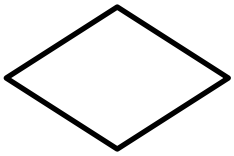
1. Write the two numbers being multiplied at the top of two separate columns.
2. Double the number in the first column and half the number in the second column. Ignore any remainders when halving.
3. If the number in the second column is even, cross out the whole row.
4. Repeat the process from step 2 until the number in the second column is 1.
5. Add up the numbers that have **not** been crossed out in the first column. This is the answer to the product.

Task 2

Use the Russian Peasant Algorithm to calculate 14 x 138

Check your answer via a different method.

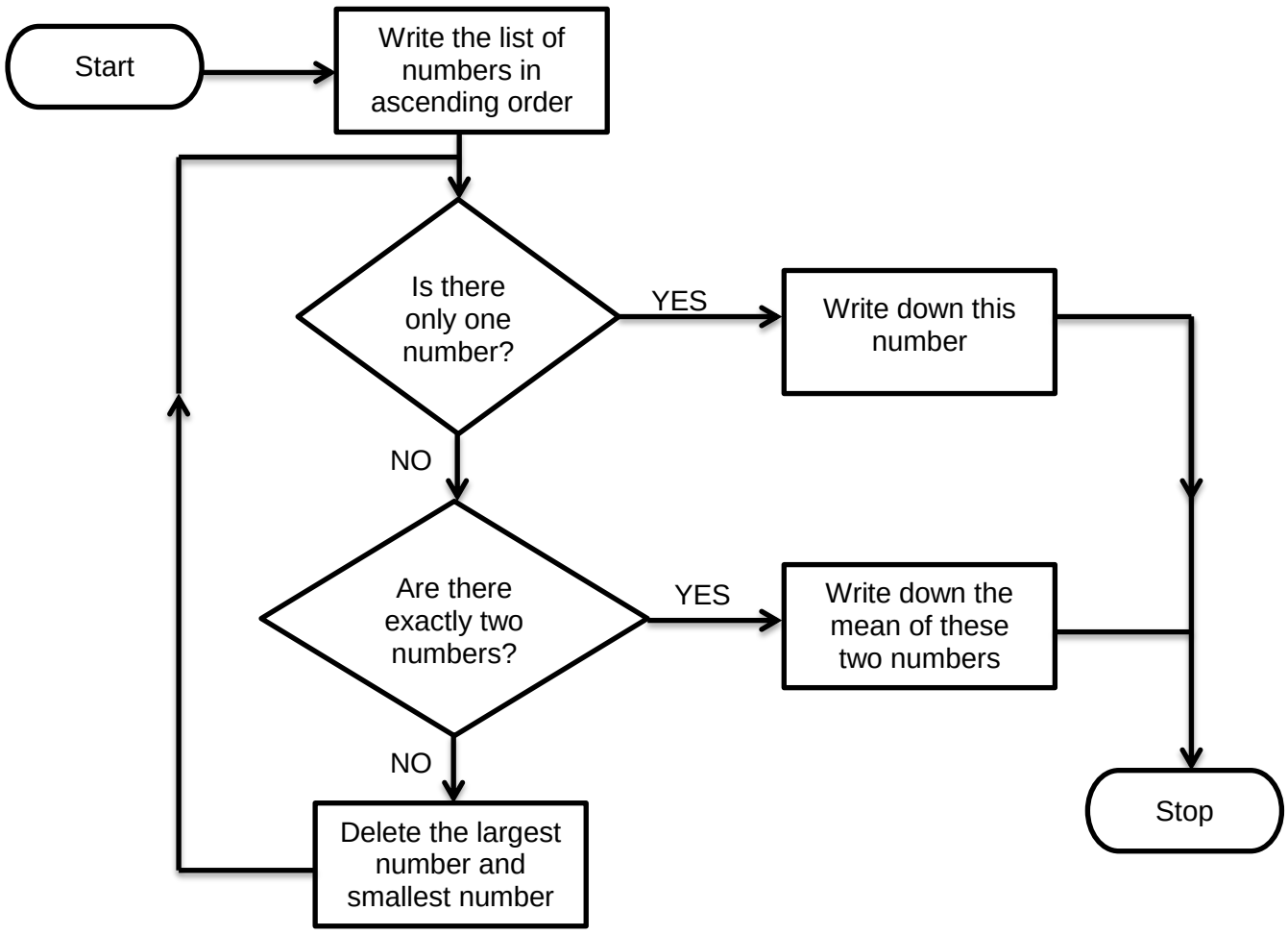
Flow Charts can be used to represent algorithms. The boxes used are:

 Rectangles contain instructions to be carried out.	 Diamonds are used for questions or decisions.
 An oval shape is sometimes used to start and finish the flow chart.	 Arrows or connectors are used to show the route through the flow chart.

Task 3

Write down a list of eight numbers of your choice.

Follow the algorithm given in then flow chart. What does the algorithm do? Try the algorithm again with a list of nine numbers. How is the length of the list relevant in then algorithm?



Task 4

Design a flow chart that illustrates the process of using the quadratic formula to solve quadratic equations. Make sure you include all possible outcomes, i.e.

- Two real distinct roots (which your flow chart should calculate);
- Two real equal roots (which your flow chart should calculate);
- No real roots.

Solution

Task 1

Suppose we wanted to know the day of the week on which the coronation of Queen Elizabeth II took place. The date of the coronation was 2nd June 1953.

1. D = 2, M = 6, Y = 1953
2. No change
3. C = 19, K = 53
4. 2.6M – 5.39 = 10.21 (integer part 10), K÷4 = 13.25 (integer part 13) and C÷4 = 4.75 (integer part 4). Sum of the integer parts = 27. Then 27 + 2 + 53 = 82. Subtract 2x19 to get 44.
5. Remainder when divided by 7 is 2
6. 2nd June 1953 was a Tuesday.

Task 2

Column 1 (Doubling)	Column 2 (Halving)
~~14~~	~~138~~
28	69
~~56~~	~~34 (remainder omitted)~~
112	17
~~224~~	~~8 (remainder omitted)~~
~~448~~	~~4~~
~~896~~	~~2~~
1792	1

So 14 x 138 = 28 + 112 + 1792 = 1932

Task 3

The algorithm finds the median. It starts by putting the numbers in ascending order and checks if the list is 1 number long (in which case we have the median). It next checks if the list has two numbers – in this case the mean of the two numbers would be the median.

If th list has more than 1 or 2 numbers, a number is crossed off each end of the list and the checks are repeated until there is only one number or two numbers in the list.

If the original list has an even number of numbers, we will end with 2 numbers in the list; if the original list has an odd number of numbers we will end with 1 number in the list.

Task 4