The Further Mathematics Support Programme

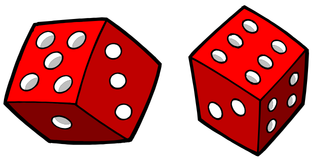
This resource is one of several available on the FMSP website: www.furthermaths.org.uk/maths-preparation

Geometric Distribution

When playing some board games, the rules state that you have to get a double 6 to start playing the game.

Suppose we decide to count the number of times (X) we have to roll the two dice before a double 6 is scored.

What is the most likely number of rolls of the two dice before we can start the game?



X is called a **discrete random variable**. It is **discrete** because it can only take fixed values and these have 'gaps' in between.

In this case the possible values of X are {1, 2, 3, 4, …} and in theory there is no upper limit on the value of X as we could keep rolling the dice forever and never get a double 6!

It is a **random** variable because the outcome is governed by chance.

So, which value of X is most likely?

If X = 1, this means that a double 6 is scored on the first roll, which has probability $\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$.

(This means that the chance of not getting a double 6 is $\frac{35}{36}$).

If X = 2, this means that a double six is scored on the second roll, and so the first roll must not have been a double six. The chance of not getting a double 6 is $\frac{35}{36}$, therefore the probability X = 2 is $\frac{35}{36} \times \frac{1}{36}$.

We can continue this pattern as shown in the table below:

No. of rolls (X) before a double 6 is obtained	Calculations
X = 1	$\frac{1}{36}$
X = 2	$\frac{35}{36} \times \frac{1}{36}$
X = 3	$\frac{35}{36} \times \frac{35}{36} \times \frac{1}{36}$
X = 4	$\frac{35}{36} \times \frac{35}{36} \times \frac{35}{36} \times \frac{1}{36}$
X = 5	$\frac{35}{36} \times \frac{35}{36} \times \frac{35}{36} \times \frac{35}{36} \times \frac{1}{36}$

Task 1

Looking at the pattern in the table above, determine a formula for the number n of rolls of the two dice until a double 6 is obtained.

Task 2

Imagine a general situation where:

X = number of trials until a success is obtained
p = probability of a successful trial
q = probability of an unsuccessful trial

(Note that p + q = 1 as a trial is either successful or unsuccessful).

What is the probability that n trials have to take place until the first success?

Task 2 describes the **Geometric distribution** which is written

$$X \sim Geo(p)$$

where p is the **parameter** of the distribution and is the probability of success.

Task 3

A darts player has a 0.1 chance of hitting the 'bullseye' in the centre of the dartboard.

If he aims for the bullseye, what is the probability that he hits it for the first time:

a) On his third dart
b) Before his third dart
c) On one of his first three darts

Task 4

Meteorologists estimate that the probability that it will rain on any day in June in Manchester is 0.35.

What is the probability that it rains on June 20th?

What is the probability that the 20th is the first rainy day in June?

Solutions

Task 1

If the first double 6 occurs on the n^{th} trial, this means that there were (n-1) unsuccessful rolls of the dice before the double 6 occurred. The probability of each unsuccessful trial is $\frac{35}{36}$ and so the probability is $\left(\frac{35}{36}\right)^{n-1} \times \frac{1}{36}$.

Task 2

In a general case, if the first success occurs on the n^{th} trial, this means there were (n-1) unsuccessful trials, each with a probability of q, before the successful trial. Hence the probability is q^{n-1} x p. This could also be written as $(1 – p)^{n-1}$ x p.

Task 3

In this case, p = 0.1 and so q = 0.9

Using the formula q^{n-1} x p we know the probability of the first bullseye on the nth trial is

0.9^{n-1} x 0.1

So the probability of obtaining a bullseye:

a) On his third dart = 0.9^2 x 0.1 = 0.081
b) Before his third dart = first or second dart = 0.1 + (0.9 x 0.1) = 0.19
c) On one of his first three darts = first, second or third dart = 0.1 + (0.9 x 0.1) + (0.9^2 x 0.1) = 0.271

Notice how important it is to read the wording in the question carefully.

Task 4

Assuming that the weather on each day is independent of the other days, the probability it rains on June 20th in Manchester is 0.35 (the same as all the other days in June!).

The probability that the first rainy day in Manchester in June is the 20th is

$(1 – 0.35)^{19}$ x 0.35 = 0.0000976

i.e. a very unlikely event!