## Further Mathematics Support Programme

### AQA AS Further Pure – Suggested Scheme of Work (2017-2018)

This template shows how **Integral Resources** and **FMSP FM videos** can be used to support Further Mathematics students and teachers. This template is for the compulsory Core Pure component of AS Further Mathematics – you will also need to deliver optional elements alongside this.

**This content makes up 50% of the AQA AS Further Mathematics content.**

It is examined in AS level Paper 1 and forms approximately ⅓ of the content examined in A level Paper 1 & Paper 2.

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<tr>
<th><strong>Integral Resources</strong></th>
<th><strong>FMSP FM videos</strong></th>
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<td><strong>Teacher access</strong> to the <strong>Integral Resources</strong> (<a href="https://integralmaths.org/2017/">integralmaths.org/2017/</a>) for Further Mathematics is available <strong>free of charge</strong> to all schools/colleges that register with the Further Mathematics Support Programme: <a href="http://furthermaths.org.uk">furthermaths.org.uk</a>. This will include access to the <strong>FM videos</strong>. A single student login will also be included so that teachers can give students direct access to the <strong>FM videos</strong>.</td>
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<tr>
<td><strong>Individual student access</strong> to the full range of <strong>Integral Resources</strong> and the <strong>FM videos</strong> for Further Mathematics is available at a cost of £30 per student or via a full school/college subscription to Integral. Teachers will get access to the management system so they can monitor their students' progress: <a href="http://furthermaths.org.uk/fm-videos">furthermaths.org.uk/fm-videos</a>.</td>
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**Integral Resources** include a wide range of resources for both teacher and student use in learning and assessment. Interactive resources and ideas for using technology are featured throughout. Sample resources are available via: [integralmaths.org/2017/](https://integralmaths.org/2017/).

**FM videos** are available for individual components of AS and A level Further Mathematics. There will be around 4-5 videos of 5-10 minutes in length for each section in Integral. The intention of these videos is that they are sufficient to introduce students to the concepts so that they can learn the material by working through appropriate examples. **FM videos** are ideal for schools/colleges teaching Further Mathematics with small groups and/or limited time allocation. They are also useful to support less experienced teachers of Further Mathematics. See [furthermaths.org.uk/fm-videos](http://furthermaths.org.uk/fm-videos).

Scheduling will depend on circumstances, but the template breaks the study into topic sections. Each section corresponds to one set of videos and may be allocated approximately equal time – this would equate to approximately one week of teaching time for a single teacher delivering the complete AS course. Further information on scheduling can be found at [furthermaths.org.uk/offering-fm](http://furthermaths.org.uk/offering-fm). FMSP Area Coordinators will be able to offer additional guidance if needed: [furthermaths.org.uk/regions](http://furthermaths.org.uk/regions).
### AQA AS Further Pure – Suggested Scheme of Work (2017-2018)

<table>
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<tr>
<th>Date</th>
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</table>
|      | **Matrices 1:** Introduction and matrix operations | • Add, subtract and multiply conformable matrices; multiply a matrix by a scalar.  
• Understand and use zero and identity matrices.  
• Calculate determinants of 2 x 2 matrices.  
• Understand and use singular and non-singular matrices; properties of inverse matrices.  
• Calculate and use the inverse of non-singular 2 x 2 matrices. | ![AQA_FM_AS_Pure](https://example.com/AQA_FM_AS_Pure) / ![Matrices and transformations](https://example.com/Matrices_and_transformations) / ![Matrices and transformations 1: Introduction to matrices](https://example.com/Matrices_and_transformations_1)  
![AQA_FM_AS_Pure](https://example.com/AQA_FM_AS_Pure) / ![Matrices and transformations](https://example.com/Matrices_and_transformations) / ![Matrices and transformations 4: Determinants and inverses](https://example.com/Matrices_and_transformations_4) | ![Exercise level 1](https://example.com/Exercise_level_1)  
![Exercise level 2](https://example.com/Exercise_level_2)  
![Section test M1](https://example.com/Section_test_M1)  
![Exercise level 3](https://example.com/Exercise_level_3) (Extension) | 1.1 Basic operations with matrices  
1.2 Multiplying matrices  
1.3 Properties of matrix multiplication  
1.4 Using zero and identity matrices  
1.5 Determinants and inverses (2x2) | Students should learn how to use their calculator to multiply matrices. | TBC  
e.g. UM, Nrich, ExamSolutions. |
|      | **Matrices 2:** Transformations and invariance | • Use matrices to represent linear transformations in 2D; successive transformations; single transformations in 3D (3D transformations confined to reflection in one of $x = 0$, $y = 0$, $z = 0$ or rotation about one of the coordinate axes) (knowledge of 3D vectors is assumed).  
• Find invariant points and lines for a linear transformation. | ![AQA_FM_AS_Pure](https://example.com/AQA_FM_AS_Pure) / ![Matrices and transformations](https://example.com/Matrices_and_transformations) / ![Matrices and transformations 2: Matrices and transformations](https://example.com/Matrices_and_transformations_2)  
![AQA_FM_AS_Pure](https://example.com/AQA_FM_AS_Pure) / ![Matrices and transformations](https://example.com/Matrices_and_transformations) / ![Matrices and transformations 3: Invariance](https://example.com/Matrices_and_transformations_3) | ![Exercise level 1](https://example.com/Exercise_level_1)  
![Exercise level 2](https://example.com/Exercise_level_2)  
![Section test M2](https://example.com/Section_test_M2)  
![Exercise level 3](https://example.com/Exercise_level_3) (Extension) | 2.1 Reflections in 2-D  
2.2 Rotations in 2-D  
2.3 Stretches, enlargements & shears in 2-D  
2.4 Successive transformations in 2-D  
2.5 Transformations in 3-D  
2.6 Invariant lines and points | Knowledge of 3-D vectors is assumed. 3-D transformations confined to reflections in $x = 0$, $y = 0$, $z = 0$ or rotation about one of the coordinate axes. |
| Complex numbers 1: Introduction | Solve any quadratic equation with real coefficients. | Exercise level 1 | 1.1 Solving quadratic equations with no real roots |
| Add, subtract, multiply and divide complex numbers in the form $x + iy$ with $x$ and $y$ real; understand and use the terms ‘real part’ and ‘imaginary part’. | Exercise level 2 | 1.2 Adding and subtracting complex numbers |
| Understand and use the complex conjugate. | Section test C1 | 1.3 Multiplying complex numbers |
| | Complex numbers 1: Introduction to complex numbers | 1.4 Dividing complex numbers |
| | Exercise level 3 (Extension) | 1.5 Terminology (real and imaginary part, conjugate) |
| | Exercise level 3 (Extension) | 1.6 Linking complex roots to quadratic equations |
| Complex numbers 2: The Argand diagram | Use and interpret Argand diagrams. | Exercise level 1 | 2.1 Introducing the Argand diagram; representing addition and subtraction |
| | | Exercise level 2 | 2.2 Representing rotation and multiplication by $i$ |
| | | Section test C2 | |
| Roots of equations | Understand and use the relationship between roots and coefficients of polynomial equations up to quartic equations. | Exercise level 1 | 1.1 Finding the relationship between roots and coefficients |
| | Form a polynomial equation whose roots are a linear transformation of the roots of a given polynomial equation (of at least cubic degree). | Exercise level 2 | 1.2 Using the relationship between roots and coefficients |
| | Know that non-real roots of polynomial equations with real coefficients occur in conjugate pairs. | Section test R1 | 1.3 Forming equations with related roots |
| | Solve cubic or quartic equations with real | Exercise level 3 (Extension) | 1.4 Forming equations with related roots by substitution |
| | | | 1.5 Solving polynomials with complex roots |
| Coefficients (given sufficient information to deduce at least one root for cubics or at least one complex root or quadratic factor for quartics). |
|---|---|---|---|

**Sequences and series 1: Summing series**
- Understand and use formulae for the sums of integers, squares and cubes and use these to sum other series.
- Understand and use the method of differences for summation of series.

**Sequences and series 2: Induction**
- Construct proofs using mathematical induction; contexts include sums of series, divisibility, and powers of matrices.
### Complex numbers 3: Modulus, argument and loci
- Convert between the Cartesian form and the modulus-argument form of a complex number (knowledge of radians is assumed).
- Multiply and divide complex numbers in modulus-argument form (knowledge of radians and compound angle formulae is assumed).
- Construct and interpret simple loci in the Argand diagram such as \(|z - a| > r\) and \(\arg(z - a) = \theta\) (knowledge of radians is assumed).

#### Exercises
- **Exercise level 1**
- **Exercise level 2**
- **Section test G1**
- **Exercise level 3** (Extension)

#### Topics
- **3.1 Cartesian to modulus-argument form**
- **3.2 Modulus-argument to Cartesian form**
- **3.3 Multiplying in modulus-argument (polar) form**
- **3.4 Dividing in modulus-argument (polar) form**
- **3.5 Modulus definitions for loci**
- **3.6 Argument definitions for loci**
- **3.7 Regions defined in the complex plane**

### Vectors and 3D Space 1: The scalar product
- Calculate the scalar product and use it to calculate the angle between two lines.
- Check whether vectors are perpendicular by using the scalar product.
- Calculate the perpendicular distance between two lines and from a point to a line.

#### Exercises
- **Exercise level 1**
- **Exercise level 2**
- **Section test V1**
- **Exercise level 3** (Extension)

#### Topics
- **1.1 Using vectors**
- **1.2: The scalar product of two**
- **1.3: The angle between two vectors**

### Vectors and 3D Space 2: The equations of a line
- Understand and use the vector and Cartesian forms of an equation of a straight line in 3D.
- Find the intersection of a line and a line.

#### Exercises
- **Exercise level 1**
- **Exercise level 2**
- **Section test V2**
- **Exercise level 3** (Extension)

#### Topics
- **The vector equation of a line in 2D**
- **2.1: The vector equation of a line in 2D.**
- **2.2: The vector equation of a line in 3D**

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Students should be familiar with radians before starting this section. Knowledge of compound angle formulae is assumed.

Students would benefit from access to a 3D graphing package.

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## Further Mathematics Support Programme

|  | • Know that two lines in three dimensions may either intersect, be parallel or be skew  
• Find the distance between a point and a line  
• Find the distance between two lines  |
|-----------------|--------------------------------------------------------------------------------|
|  | 2.3: The equation of a line in 3D in Cartesian form  
2.4: The intersection of lines in 2D  
2.5: The intersection of lines in 3D  
2.6: The perpendicular distance from a point to a line  
2.7: The perpendicular distance between two parallel  
2.8: The perpendicular distance between skew lines  |

### Rational Functions

1: Graphs of rational functions

- Graphs of rational functions of form \( \frac{ax+b}{cx+d} \); asymptotes, points of intersection with coordinate axes or other straight lines; associated inequalities.
- Graphs of rational functions of form \( \frac{ax^2+bx+c}{dx^2+ex+f} \), including cases when some of these coefficients are zero; asymptotes parallel to coordinate axes.
- Using quadratic theory (not calculus) to find the possible values of the function and coordinates of the stationary points of the graph for rational functions of form \( \frac{ax^2+bx+c}{dx^2+ex+f} \).

### Exercise Level 1

- Exercise level 1
- Exercise level 2
- Section test RF1
- Exercise level 3 (Extension)

### Further Resources

- AQA_FM_AS_Pure
- Vectors and 3D space
- Vectors and 3D space: Topic assessment

Students would benefit from access to graphing software.
## Rational Functions 2: Inequalities
- Inequalities involving polynomial equations (cubic and quartic).
- Solving inequalities such as \( \frac{ax+b}{cx+d} < ex+f \) algebraically.

### Inequalities
- Exercise level 1
- Exercise level 2
- Section test RF2
- Exercise level 3 (Extension)

### Students would benefit from access to graphing software.

## Algebra & graphs 3: Conic sections / polar curves
- Sketching graphs of curves with equation \( y^2 = 4ax, \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1, xy = c^2 \), including intercepts with axes and equations of asymptotes of hyperbolas.
- Single transformations of curves involving translations, stretches parallel to coordinate axes and reflections in the coordinate axes and the lines \( y = \pm x \).

### Conics
- Exercise level 1
- Exercise level 2
- Section test CS1

### Students would benefit from access to graphing software.

## Sequences and series 3: Maclaurin series
- Recognise and use the Maclaurin series for \( e^x, \ln(1 + x), \sin x, \cos x, \) and \( (1 + x)^n \), and be aware

### Maclaurin approximations and series
- Exercise level 1
- Exercise level 2
- Section test S3

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<td>of the range of values of $x$ for which they are valid (proof not required).</td>
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<tr>
<td>Hyperbolic functions</td>
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<td>Calculus: Volumes of revolution</td>
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Consolidation and revision