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A farmer has 240m of fencing and wants to create an enclosed area for his sheep. What is the biggest area that he can fence off?



The starting point is to consider rectangular areas. If we let one side be x , the other side will be $120 - x$. The area of the rectangle would therefore be $x(120 - x)$.

The problem is therefore to maximise $x(120 - x)$. This can be done either experimentally or analytically.

One way of doing this analytically is to complete the square.

$$A = 120x - x^2$$

$$A = -(x^2 - 120x)$$

$$A = -((x - 60)^2 - 3600)$$

$$A = 3600 - (x - 60)^2$$

The maximum would therefore be when the side length is 60 m giving an area of 3600 m^2

This is a square!

The next thing the students may wish to consider is the possibility of a different shape giving a larger enclosure.

Using a regular hexagon with side length 40 m which can be split into 6 equilateral triangles (each of side 40 m) gives an area of $6 \times \frac{1}{2} \times 40 \times 40 \times \sin 60 = 2400\sqrt{3}\text{ m}^2$ this is more than 3600 m^2 .

This eventually should lead to considering a circle with circumference 240 m .

The radius of the circle would be $\frac{240}{2\pi} = \frac{120}{\pi}$ and the area would be $\pi \times \left(\frac{120}{\pi}\right)^2 = \frac{14400}{\pi} = 4584\text{ m}^2$ to the nearest whole number.