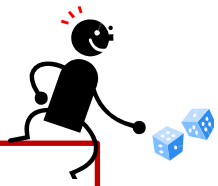
The Further Mathematics Support Programme

Decision Tree Group Tasks

For each of the tasks below, produce a decision tree and answer the questions which follow.

Task A



You have a choice of two games to play.

Game 1

You pay £1.50 to play the game and roll a fair dice.

If you roll a 6 you win £30.

Otherwise you have to pay an additional £5.

Game 2

You pay £1 to play the game and roll a fair dice.

If you get a 5 or a 6 you win £3.

If you get a 3 or a 4 you win £1.

Otherwise you win nothing.

Which game would you choose (if any)? Explain why.

Task B

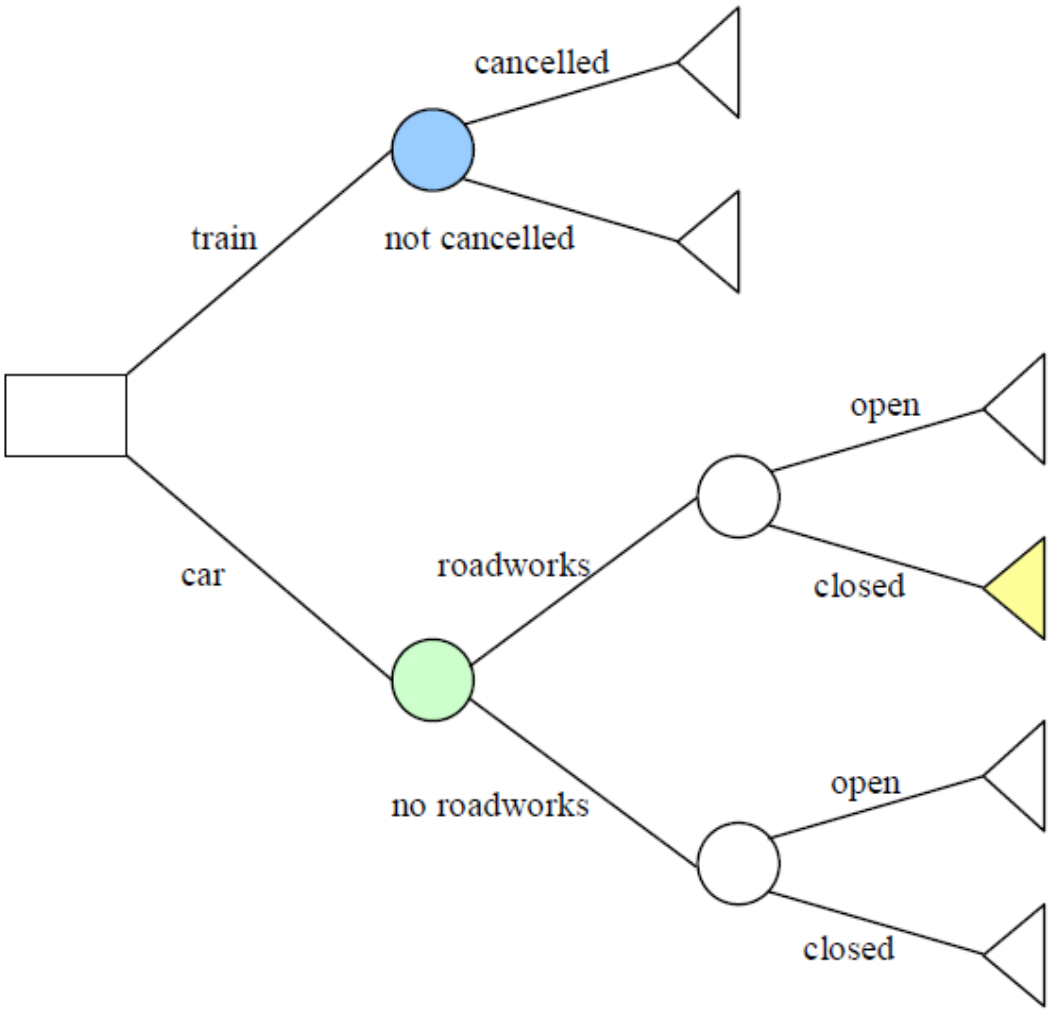
Justin can go to work by either train or by car, and wants to make the journey as short as possible.

If he goes by train the journey takes 20 minutes, but occasionally the train is cancelled and he has to wait 30 minutes for the next one. The probability that the train is cancelled on a particular day is 0.02.

If he goes by car the journey usually takes 15 minutes. However, sometimes there are roadworks, which adds 10 minutes to the journey, and sometimes the level crossing is closed, which adds 3 minutes to the journey. The probability that there are roadworks is 0.15, and the probability that the level crossing is closed at the time Justin goes past is 0.4.

Complete the decision tree on the next page and determine what is the best strategy by which Justin should travel to work.

Task B (continued)



Task C

Neelam has been offered the opportunity to invest in a new business.

If the business is a success, she will make a profit of £5000, if it fails she will lose £2000.

Data on similar businesses that have been started in the past shows that there is a 60% chance that the business will succeed.

Neelam has the opportunity to have the business plan examined by an expert who will make a report on it, for a fee of £400. If the expert's report is favourable then there is an 80% chance that the business will succeed; if the report is unfavourable then there is a 20% chance that the business will succeed.

In the past the expert has given favourable reports in 70% of cases.

(i) Draw a decision tree and use it to find Neelam's optimal decision.

(ii) What is the most that Neelam should be prepared to pay for the expert's report?

Task D

A company has developed a new product and now need to decide whether to launch the product straight away, run some test marketing or not launch the product at all.

If the product is launched straight away, market research suggests strong sales with a probability of 0.3, leading to profit of £200,000. Weak sales would occur with probability 0.7 and lead to a profit of £50,000.

To run test marketing would cost £15,000 which will be deducted from profits. The marketing will give a positive or negative indication about the market, and both are equally likely. If the marketing is positive then the probability of a strong market is 0.8. If the marketing is negative the probability of a weak market is 0.8.

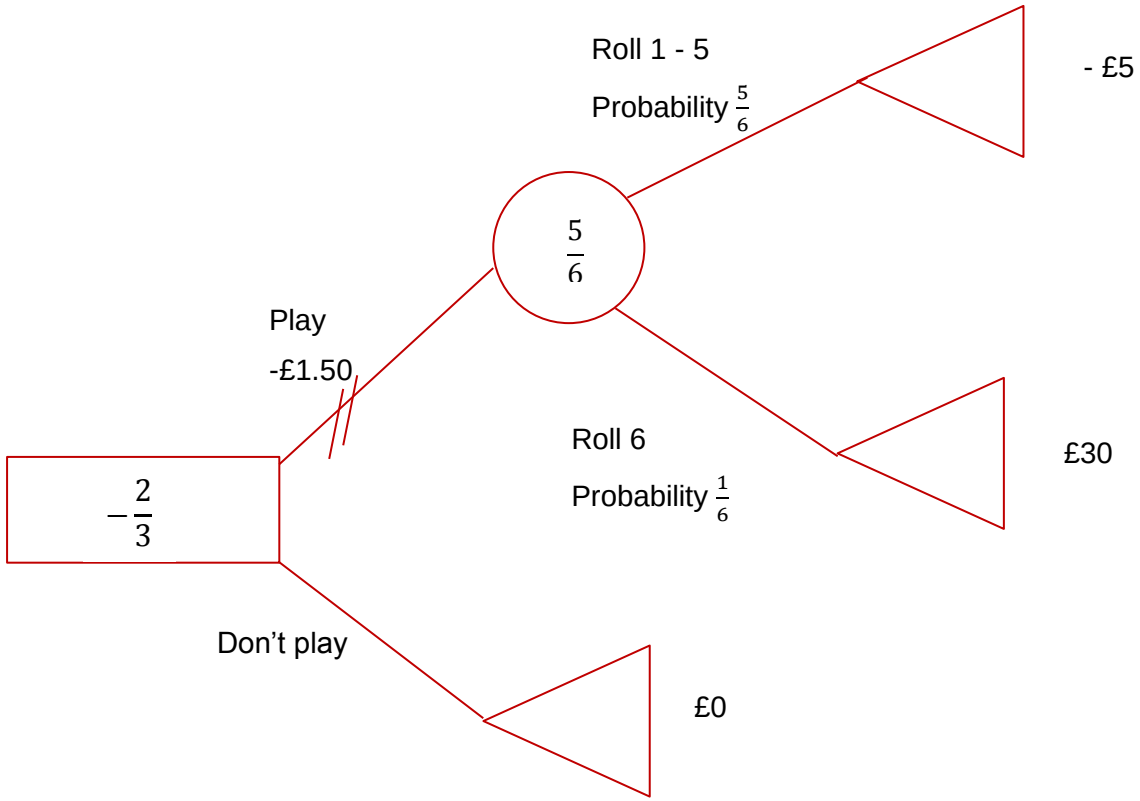
The company can decide not to launch the product either before or after the test marketing.

If not launched, the product could be sold to a rival company for £90,000.

Draw a decision tree and use it to advise the company what they should do.

Task A

Game 1: You pay £1.50 to play the game and roll a fair dice. If you roll a 6 you win £30. Otherwise you have to pay an additional £5.

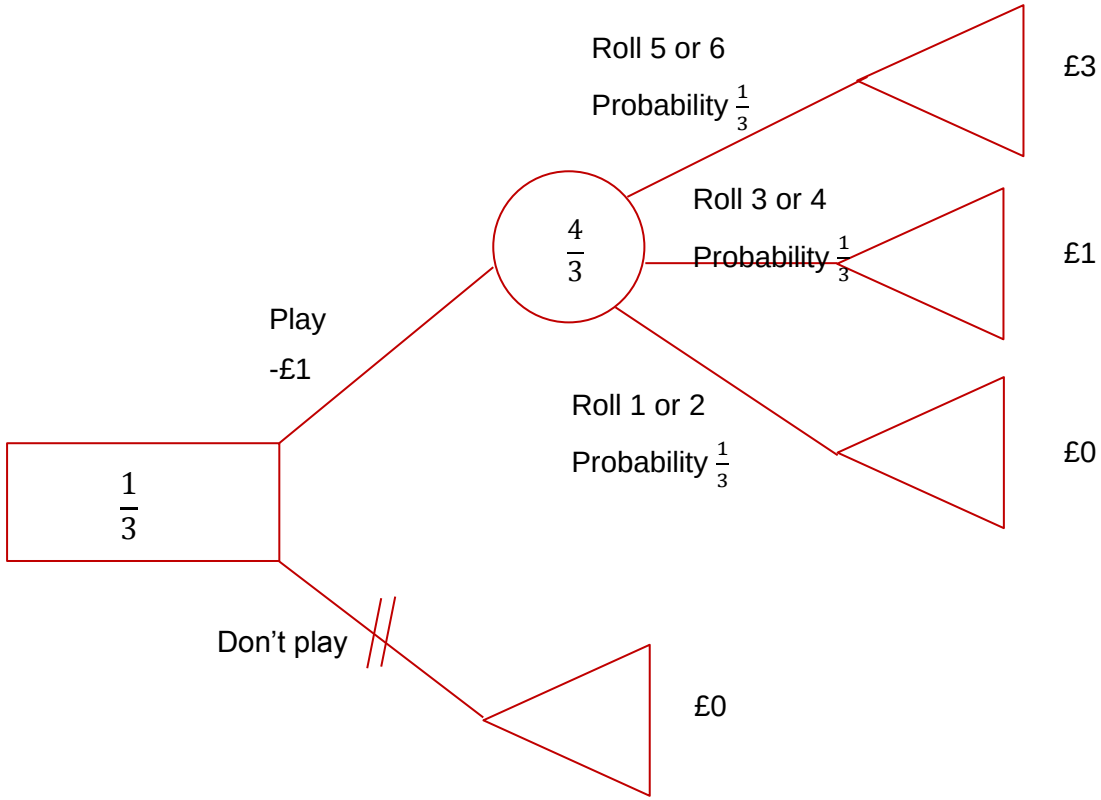


For top branches, $\left(\frac{5}{6} \times -£5\right) + \left(\frac{1}{6} \times £30\right) = \frac{5}{6}$

But this EMV needs to take into account the £1.50 cost to play the game, so the overall EMV is $\frac{5}{6} - 1.50 = -\frac{2}{3}$ i.e. a loss of approximately 67p

So choose not to play Game 1.

Game 2: You pay £1 to play the game and roll a fair dice. If you get a 5 or a 6 you win £3. If you get a 3 or a 4 you win £1. Otherwise you win nothing.



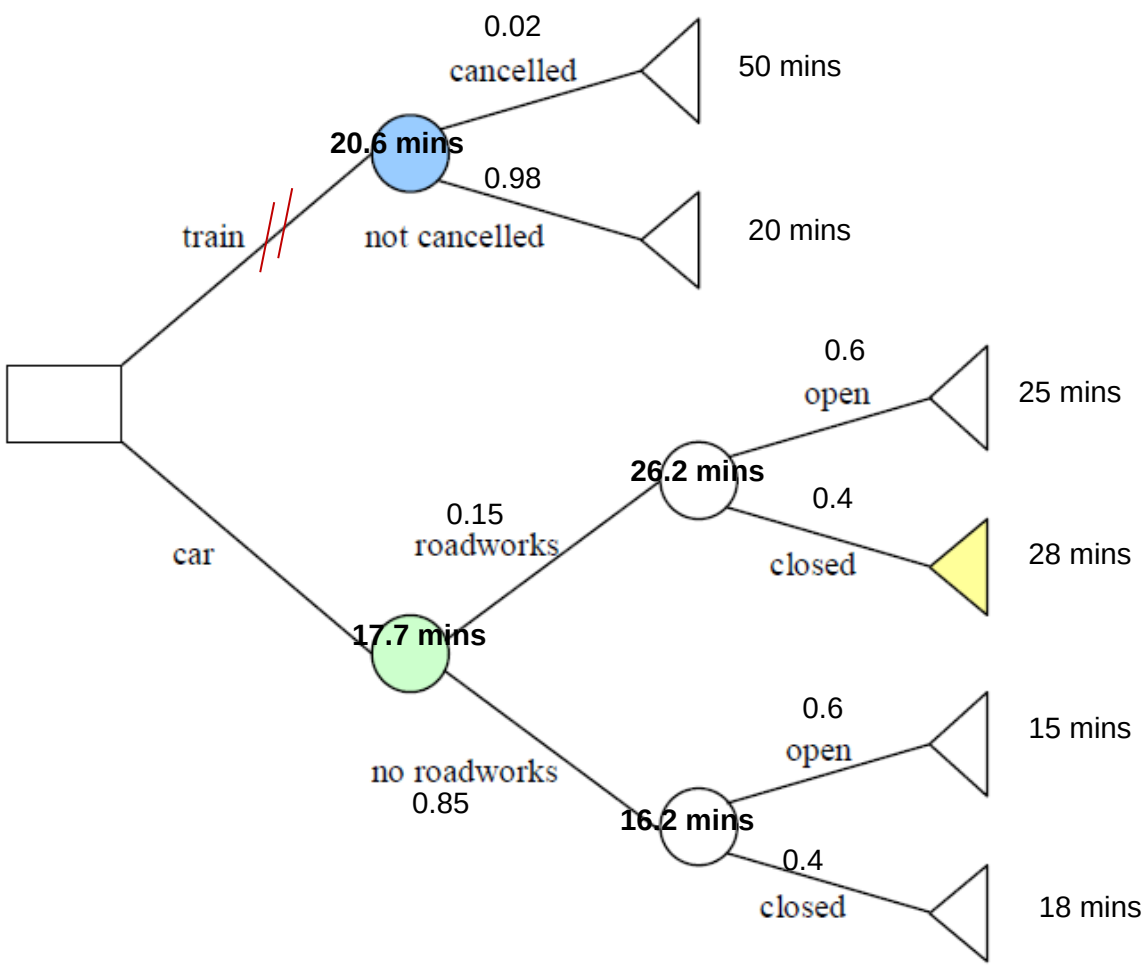
For top branches, $\left(\frac{1}{3} \times £3\right) + \left(\frac{1}{3} \times £1\right) + \left(\frac{1}{3} \times £0\right) = \frac{4}{3}$

But this EMV needs to take into account the £1 cost to play the game, so the overall EMV is $\frac{4}{3} - 1 = \frac{1}{3}$ i.e. a win of approximately 33p

So choose to play Game 2.

Overall, choose to play Game 2, but do not play Game 1. EMV of this course of action is approximately 33p per game.

Task B



Justin can go to work by either train or by car, and wants to make the journey as short as possible.

If he goes by train the journey takes 20 minutes, but occasionally the train is cancelled and he has to wait 30 minutes for the next one. The probability that the train is cancelled on a particular day is 0.02.

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Solution:

Train: (0.02 x 50) + (0.98 x 20) = 20.6 mins

Car roadworks: (0.6 x 25) + (0.4 x 28) = 26.2 mins

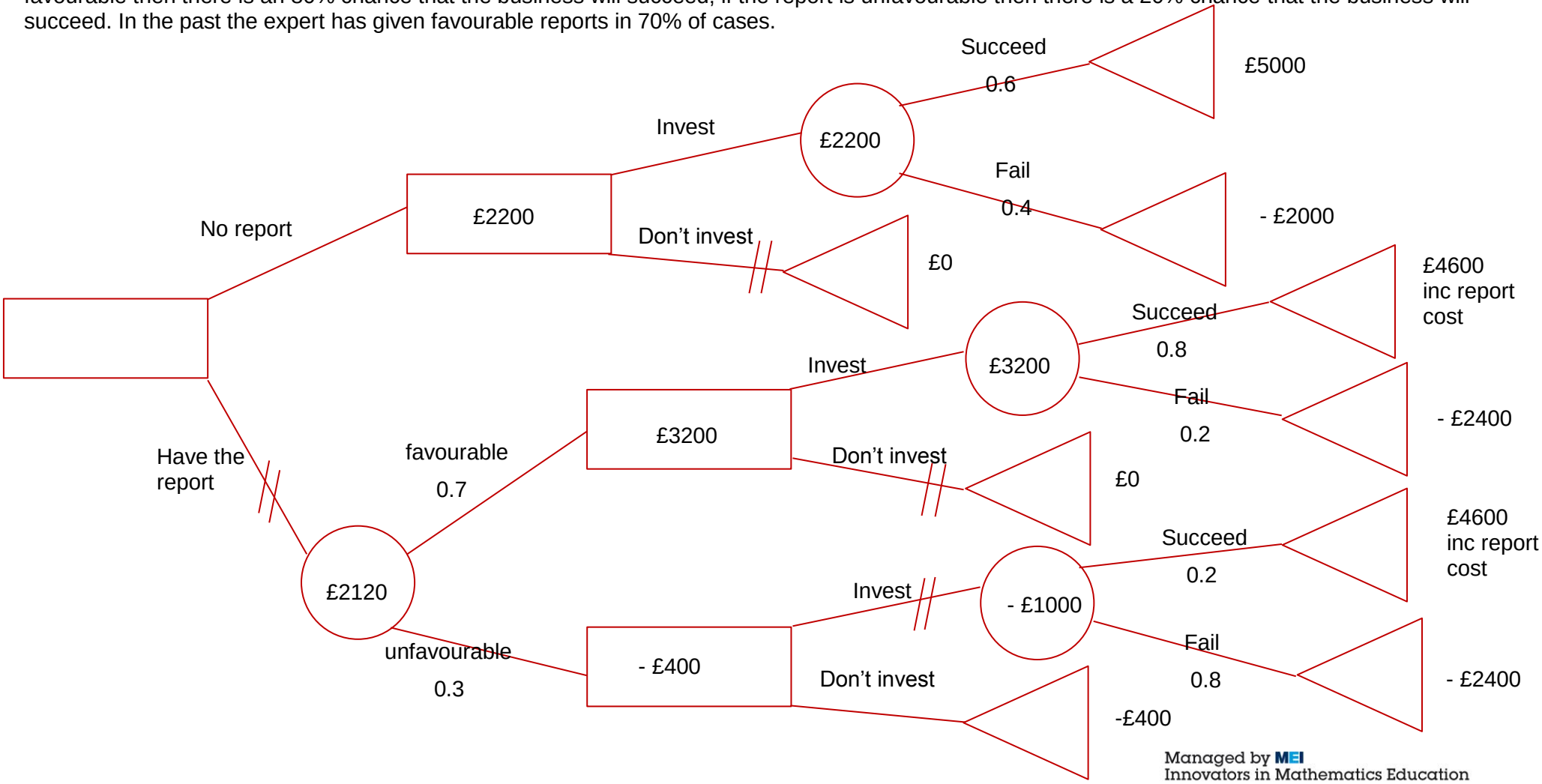
Car no roadworks: (0.6 x 15) + (0.4 x 18) = 16.2 mins

Car overall: (0.15 x 26.2) + (0.85 x 16.2) = 17.7 mins

So lowest expected travel time is 17.7 mins and Justin should take the car.

Task C

Neelam has been offered the opportunity to invest in a new business. If the business is a success, she will make a profit of £5000, if it fails she will lose £2000. Data on similar businesses that have been started in the past shows that there is a 60% chance that the business will succeed. Neelam has the opportunity to have the business plan examined by an expert who will make a report on it, for a fee of £400. If the expert's report is favourable then there is an 80% chance that the business will succeed; if the report is unfavourable then there is a 20% chance that the business will succeed. In the past the expert has given favourable reports in 70% of cases.



Task C (continued)

(i) Working backwards, right to left.

No report: (0.6 x £5000) + (0.4 x -£2000) = £2200 which is better than not investing.

Report (favourable): (0.8 x £4600) + (0.2 x -£2400) = £3200 which is better than not investing

Report (not favourable): (0.2 x £4600) + (0.8 x -£2400) = - £1000 which is worse than not investing.

Overall, for report: (0.7 x £3200) + (0.3 x -£400) = £2120

£2120 is less than £2200 so best course of action is to invest without having a report. This has EMV of £2200.

(ii) Having a report has EMV which is only £80 worse than not having a report. Hence if the report was £320 or less it would be worth carrying out, as it would yield an EMV equal to or better than not having the report.

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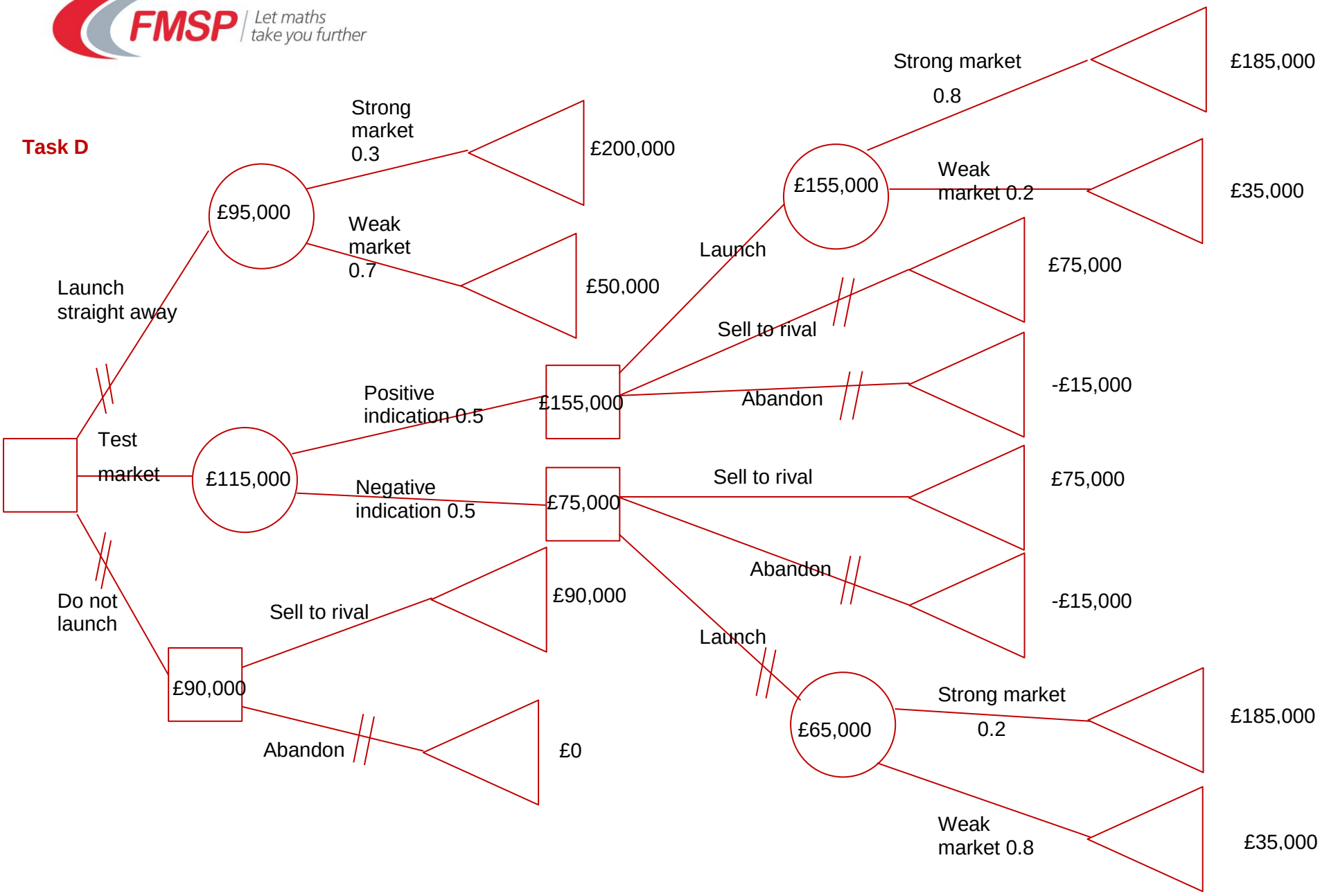
Task D



Task D (continued)

Moving from right to left in the decision tree:

Test market, positive indication, launch: (0.8 x £185,000) + (0.2 x £35,000) = £155,000 so best option is to launch.

Test market, negative indication, launch: (0.2 x £185,000) + (0.8 x £35,000) = £65,000 so best not to launch and sell to competitor.

Test market overall: (0.5 x £155,000) + (0.5 x £75,000) = £115,000.

Do not launch branch: best to sell to rival for £90,000 than abandon.

Launch straight away branch: (0.3 x £200,000) + (0.7 x £50,000) = £95,000

So best course of action is to do a test marketing exercise and if the indication is negative, sell to rival; if indication is positive go ahead to launch. The EMV of this plan of action is £115,000.