

Trigonometry: The Willis Tower

For the base section (2 cards)

$$\text{Height} = 378.84 - 180.7 = 198.14m$$

For section 2 (1 card)

$$\text{Angle of elevation of top of base section 1} = \tan^{-1}\left(\frac{198.14}{1000}\right) = 11.207..^\circ$$

$$\text{Angle of elevation of top of section 2} = 8.5 + 11.207.. = 19.707..^\circ$$

$$\text{Height of top of section 2} = 1000 \times \tan 19.707.. = 358.20m$$

$$\text{Height of section 2} = 358.20 - 198.14 = 160.06m$$

There is an interesting conversation to be had here about the difference in “width” between tower 1 and 2 and how much this would affect the answer with observations from 1 km away.

For section 3 (1 card)

$$\text{Height} = 160.06 \div 2 = 80.03m$$

For section 4 (1 card)

$$\text{Height} = 4m$$

For the TV masts (3 cards)

$$\text{The taller mast is } \sqrt{14.14^2 - 10^2} = 10.00m \text{ taller than the second mast}$$

$$\text{The taller mast is } 75.16 + 10 = 85.18m \text{ tall}$$

$$\text{Total pinnacle height} = 358.20 + 80.03 + 4 + 85.18 = 527.41m$$

For the calculations only 8 cards are necessary. It can be argued that some of the other cards are needed to define the problem correctly.