

## Application of A level Mathematics and Further Mathematics

This application makes use of the following:

**Topics from AS level Further Mathematics**

- Forming a 2x2 matrix from a pair of simultaneous equations
- Finding the inverse of a 2x2 matrix.

### Mesh Currents

An electrical network can be modelled by a set of simultaneous equations which can be represented and solved efficiently using matrix methods.

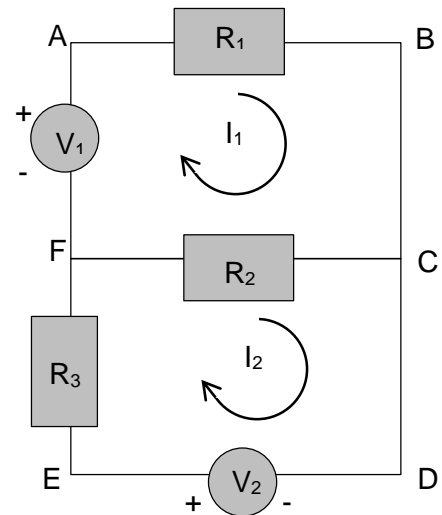
A **mesh** loop in an electrical network is a loop that does not contain any smaller loops.

The diagram below shows two meshes: ABCF and CDEF.

The mesh currents  $I_1$  and  $I_2$  are shown as running clockwise.

The net current in a branch of the network is called the **branch current** and these are found by combining the mesh currents.

For example the branch current from C to F is  $I_1 - I_2$ .



**The problem:** Calculate the mesh currents,  $I_1$  and  $I_2$ .

### Setting up the model:

**Kirchhoff's voltage law** states that the sum of voltages around a mesh is zero. This means that the voltage rises are equal to the voltage drops across a resistor, where  $V = IR$ .

Using Kirchhoff's Law for each mesh loop you can form two equations in  $V$ ,  $I$  and  $R$ .

Equation 1 for mesh ABCF:  $V_1 = I_1 R_1 + (I_1 - I_2)R_2 = I_1 (R_1 + R_2) - I_2 R_2$

Equation 2 for mesh CDEF:  $V_2 = I_2 R_3 + (I_2 - I_1)R_2 = -I_1 R_2 + I_2 (R_2 + R_3)$

The information can be written as a matrix equation of the form  $\mathbf{RI} = \mathbf{V}$ , where  $\mathbf{R}$  is a 2x2 matrix, and  $\mathbf{I}$  and  $\mathbf{V}$  are vectors.

$$\begin{pmatrix} (R_1 + R_2) & -R_2 \\ -R_2 & (R_2 + R_3) \end{pmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Solving this matrix equation involves finding the inverse of the 2x2 matrix.

**Task 1**

For a particular electric network the voltages  $V_1 = 5\text{V}$  and  $V_2 = 10\text{V}$  and the resistances  $R_1 = 3\Omega$ ,  $R_2 = 8\Omega$ , and  $R_3 = 6\Omega$ .

Substitute these values into the matrix equation and simplify.

**Task 2**

Solve the matrix equation by finding the inverse of matrix  $\mathbf{R}$ .

Show that the mesh currents  $I_1$  and  $I_2$  are equal.

Hence find the branch current between C and F.

**Solutions****Task 1**

Using values given  $\mathbf{R} = \begin{pmatrix} 11 & -8 \\ -8 & 14 \end{pmatrix}$

Therefore  $\begin{pmatrix} 11 & -8 \\ -8 & 14 \end{pmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \end{bmatrix}$

**Task 2**

$$\mathbf{R}^{-1} = \frac{1}{218} \begin{pmatrix} 14 & 8 \\ 8 & 11 \end{pmatrix}$$

Solving the matrix equation

$$\begin{aligned} \frac{1}{218} \begin{pmatrix} 14 & 8 \\ 8 & 11 \end{pmatrix} \begin{pmatrix} 11 & -8 \\ -8 & 14 \end{pmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} &= \frac{1}{218} \begin{pmatrix} 14 & 8 \\ 8 & 11 \end{pmatrix} \begin{bmatrix} 5 \\ 10 \end{bmatrix} \\ \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} &= \frac{1}{218} \begin{bmatrix} 14 \times 5 + 8 \times 10 \\ 8 \times 5 + 11 \times 10 \end{bmatrix} \\ \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} &= \frac{1}{218} \begin{bmatrix} 150 \\ 150 \end{bmatrix} \end{aligned}$$

Therefore  $I_1 = 0.69\text{A}$  and  $I_2 = 0.69\text{A}$ .

Hence the branch current between C and F =  $I_1 - I_2 = 0$ .

This example is of a simple electrical network. More complex electrical networks can be analysed using larger matrices.