The Further Mathematics Support Programme

Modelling Bacterial Growth

The population growth of bacteria can be modelled in a fairly simple way under certain circumstances. In a controlled environment where they are given sufficient nutrients and where the density of the bacteria is relatively low, the population will increase very quickly through asexual division of one cell into two. Under this model, it can be assumed that the time taken for a bacterium to mature and divide is roughly constant and the same for all bacteria.

In a laboratory the population density of bacteria can be measured by passing a beam of light through them and noting the amount that has been absorbed. More bacteria will give rise to more light being absorbed. Hence the amount of bacteria is proportional to the amount of light absorbed. So the bacterial density is expressed in units of absorbance.

In one such experiment, the population density was recorded every 10 minutes and following data resulted:

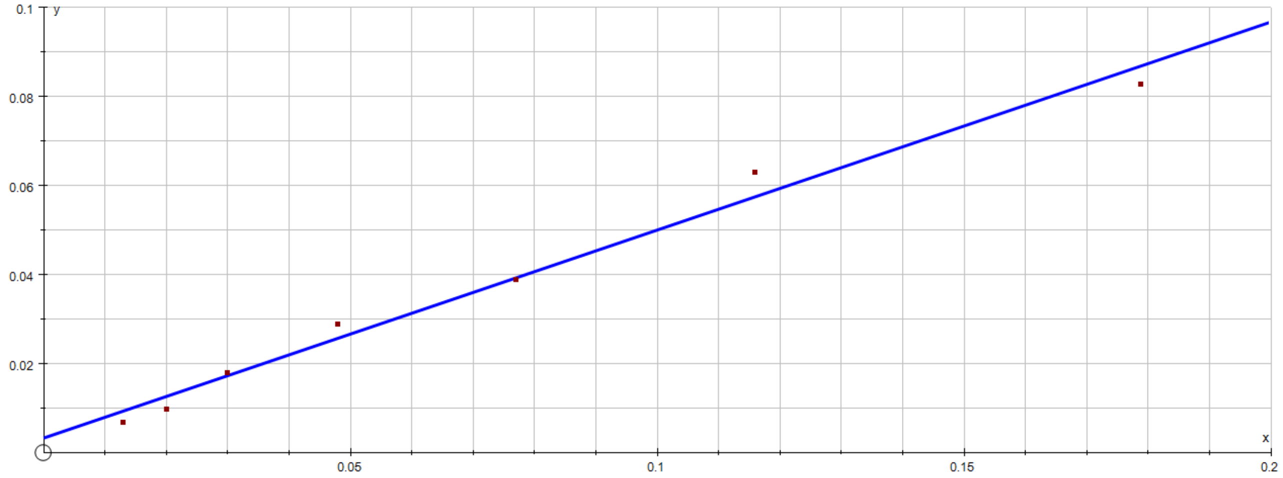
Time(Min)	Pop Density
0	0.013
10	0.020
20	0.030
30	0.048
40	0.077
50	0.116
60	0.179
70	0.262

So how might we model this data? One technique Biologists use is to look at the relationship between the change in population density and the population density, as they look for a model where the change in population is proportional to the current population.

If we do this, we get the following data:

Pop Density	Change
0.013	0.007
0.020	0.010
0.030	0.018
0.048	0.029
0.077	0.039
0.116	0.063
0.179	0.083

We can plot this data, with population density on the x-axis and change on the y-axis:



We can see from the graph that the data is close to a straight line, almost through the origin and so measuring the gradient of this line, we have the approximate relationship:

$y = 0.52x$ where 0.52 is the gradient of the line

So:

Change in Population Density = 0.52 x Population Density.

This reasonably straightforward relationship can help us to construct a function for the population at any given time.

If we let P_t stand for the density of the population at time t, then we know that:

$P_{t+1} - P_t = 0.52 P_t$

Where t stands for intervals of 10 minutes, so t = 0,1,2,3…… are all 10 minute intervals

If we rearrange this we have:

$P_{t+1} = 1.52\ P_t$, which is known as a recurrence or iterative relationship.

In particular it means that:

$P_1 = 1.52\ P_0 \quad = 1.52 \times 0.013$

$P_2 = 1.52\ P_1 \quad = 1.52^2 \times 0.013$

$P_3 = 1.52\ P_2 \quad = 1.52^3 \times 0.013$

Generalizing we get:

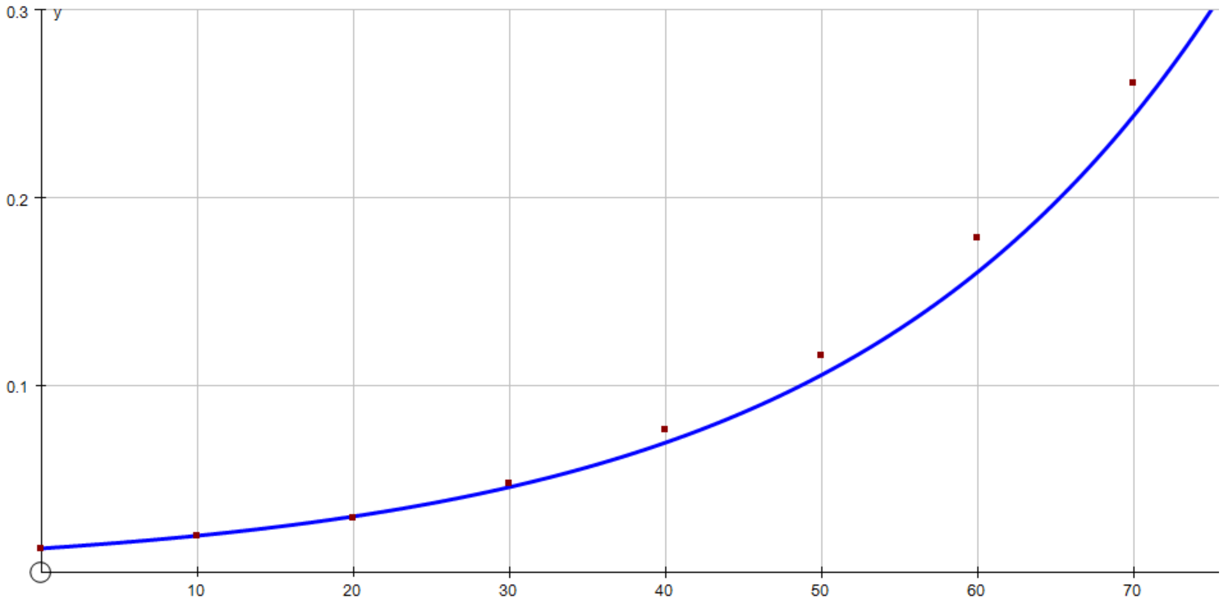
$P_t = 1.52^t \times 0.013$

And so we have a model for the population at time t = 1, 2, 3,…

However these are not the actual times but intervals of 10 minutes, so we must amend this to

$P(t) = 0.013 \times 1.52^{(t/10)}$ which gives us an estimate of the population at any time t.

If we compare this with the original data:



The model fits quite well initially but will become poorer as time increases. As the population increases there will come a point where the nutrients are no longer freely available and the model no longer applies.

References

Adapted from:

Cornette JL, Ackerman RA, and Nykamp DQ, "Developing an initial model to describe bacteria growth." From *Math Insight*. **http://mathinsight.org/bacteria_growth_initial_model** [Accessed: 26/1/15]