# Problems for the classroom

These are a selection of problems which have been created for use in the classroom. They cover different aspects of the curriculum and can be used as extension or enrichment questions, as revision of previously covered topics or as an engaging way of introducing new topics.

In addition to developing problem solving skills, these activities are linked to the following GCSE curriculum topics:

| Problem | Topics used |
|---|---|
| At the Pool | Pythagoras; rounding |
| Folded Rectangle | Area |
| The New Employee | Averages |
| Not my cup of tea | Probability and listing |
| Pentagonal desks | Interior and exterior angles |
| Pentagonal mirror tiles | Interior and exterior angles |
| Railway tracks | Estimation; approximation; scales |
| Rectangles inside a square | Area; perimeter; surds |
| Swimming | Ratios |

# At the Pool

My local swimming pool is rectangular and the dimensions are in the ratio 2:1. Normally I swim 20 lengths of the pool, however for a change I decide to swim diagonally between opposite corners.
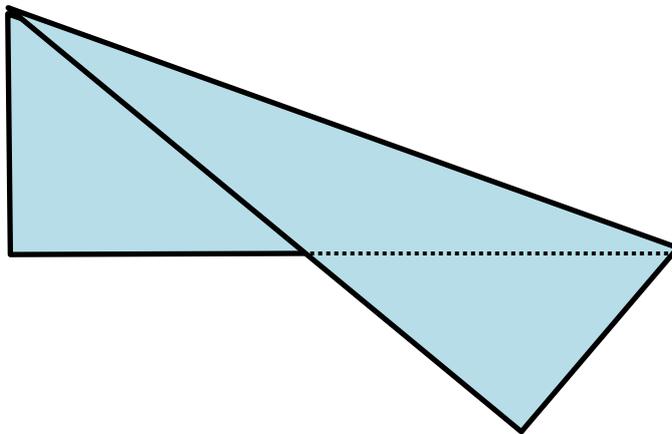
How many "diagonals" do I need to swim so that the distance is equivalent to 20 lengths?

What if the dimensions of the pool were 20m by 7.5m?

# Folded Rectangle

A rectangle of sides *a* and *b* is folded along its diagonal.



What is the area of the new shape formed?

**Further investigation**

Investigate what happens to the area of the resulting shape as the ratio of *a* : *b* changes.

# The New Employee

A small company has 4 employees. There is to be a new employee appointed and the manager will decide on their salary.

The salaries of the current employees are

Employee A = £26 000

Employee B = £40 000

Employee C = £38 000

Employee D = £35 000

Employee E = ????

A. What could the new employee be paid so that one of the current employees would be paid the modal salary?

B. What could the new employee be paid so that one of the employees would be paid the median salary?

C. What could the new employee be paid so that 4 out of 5 employees would be paid above the mean salary?

D. What could the new employee be paid so that one of the employees would be paid the mean salary?

# Not my cup of tea

Arube likes her tea with just milk.

Barbara likes her tea with milk and one sugar.

Chris likes his tea with milk and two sugars.
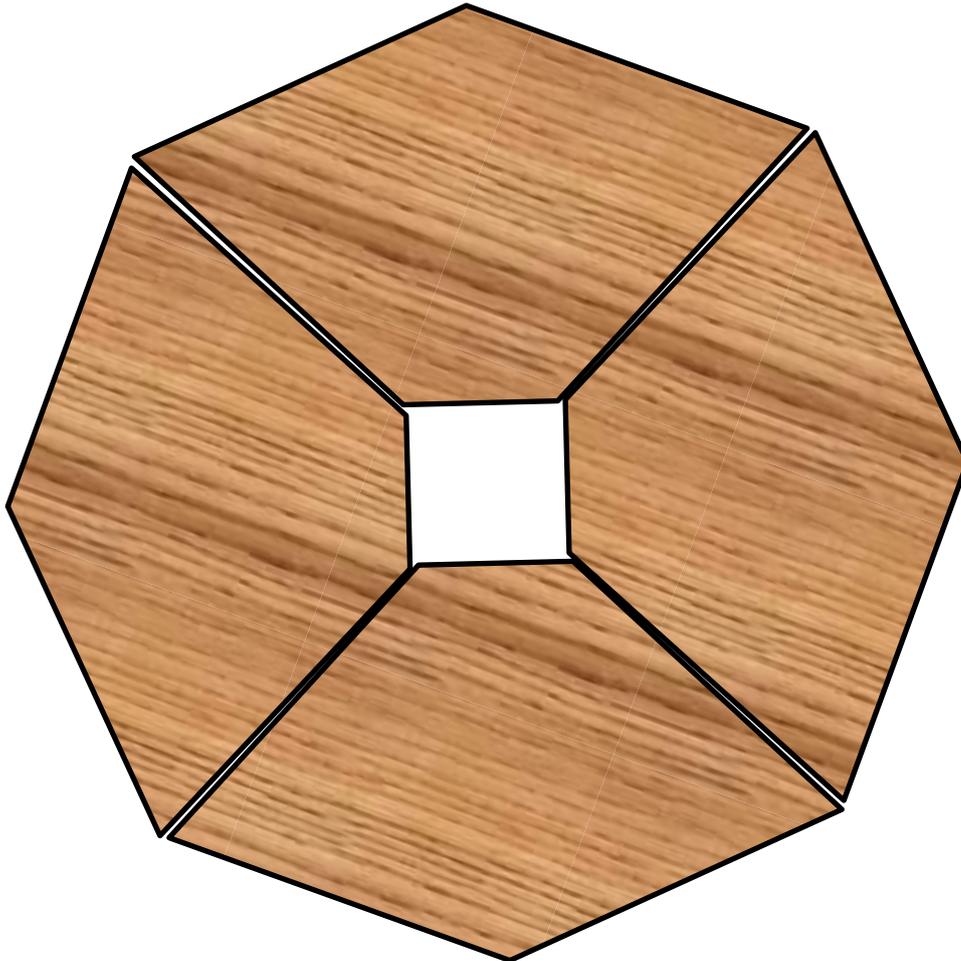
Darnel likes his tea with milk and sweetener.

On one occasion someone makes the four people their tea but gets the cups mixed up.

What is the probability that no one gets the correct cup of tea?

# Pentagonal desks

These desks have five sides and can be arranged together to make an octagon with a square hole at the centre.



Find the angles of the desks.

# Pentagonal Mirror Tiles

A mirror in the shape of a pentagon is surrounded by 5 congruent tiles.



Find the angles of the tiles.

# Railway Track

In the UK the distance between rails is called Standard Gauge and is 1435mm. The rails are held in place by wooden sleepers which the metal rails are fixed to. In the UK there are approximately 16300 km of railway track.
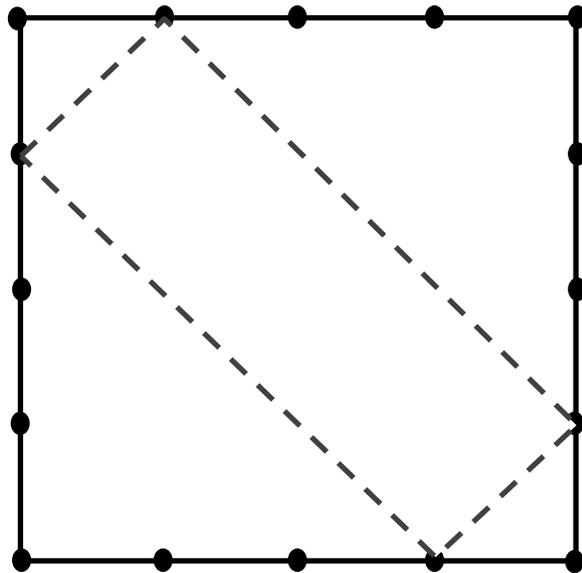


Using the photograph above estimate the number railway sleepers used in the UK railway network.

# Rectangles inside a square

The square below has sides of length = 4cm. Its perimeter is marked at equal intervals of 1cm by dots.

By joining 4 dots with straight lines, rectangles can be drawn inside the square.



- What is the longest perimeter a rectangle could have?
- What is the shortest perimeter a rectangle could have?

**Other ideas**

- What are the areas of rectangles which can be drawn?
- What other shapes can be drawn by joining the dots – can you find their areas and perimeters?

Suppose the corners of the rectangles drawn inside must touch the 4 sides of the square but do not have to be on the dots. Find how the perimeter of the rectangle will vary?

# Swimming

I go swimming with a friend, Carl, who swims faster than me.

We set off together at one end of the pool.



How much faster than me is Carl swimming if I meet him half way up the pool as he is swimming back?

What if I meet him a third the way up the pool?
What if I meet him two thirds the way up the pool?

If Carl swims four times faster than me, where do we meet?

Find a general formula for how far up the pool we would meet if Carl swims $k$ times faster than me.

**Solutions:**

**At the Pool**

I need to swim 19 diagonals (18 is just not enough)

If the dimensions were 20m by 7.5m I would need to swim 19 diagonals

**Folded Rectangle**

Area is $\frac{3}{4}ab$

**The New Employee**

A    Any of the existing salaries

B    Any amount – there will be 5 employees so one has to be the median

C    £35 000

D    £34 750

**Not my cup of tea**

$\frac{9}{24}$

**Pentagonal desks**

Three angles of 135°

Two angles of 67.5°

**Pentagonal Mirror Tiles**

Two angles of 90°

Two angles of 126°

One angle of 108°

**Railway Track**

If there are approximately2 sleepers per 1m then there are approximately 32.6 million sleepers

**Rectangles inside a square**

Longest perimeter: 16

Shortest perimeter: 10

**Swimming**

Meeting half way up: 3x faster

Meeting a third: 5x faster

Meeting two thirds: 2x faster

Swimming 4x faster we meet $\frac{2}{5}$ of the way up

General formula if K times faster:

$\frac{2}{(k+1)}$