Fractals – Notes

This session is designed to last about 50 minutes. The timings are approximate and will vary from group to group. You may find it easier to change the 'minutes elapsed' to the actual times of your presentation.

Required Knowledge:

- Students will need some knowledge of working with fractions, and looking for general terms of a sequence.
- Students will need knowledge of Pascal's triangle for the extension, however for most groups this is unlikely to fit into the time allocated for the lesson.

Resources:

- PowerPoint.
- Student work book.
- Students may wish to have coloured pencils.
- Resource sheets of triangular paper.

Objectives of session:

- To understand what is meant by a fractal.
- To investigate some fractals, and look for patterns in them.
- To discuss what happens to these fractals as we increase the number of iterations.

Time	**Activities/Questions/Points to make**	**Resources**
	Starter: Start by showing the definition of a fractal. Show the students the pictures of fractals in nature. How do these match the definition of a fractal? Students can write the definition of a fractal in their workbook.	Workbook. P2
5 min	Show the students the two pictures of fractals in geometry. For each image ask the students to think about how this matches the definition of a fraction. How are they made? Slide 12 is a Menger Sponge Slide 13: This image is the Sierpinski triangle. This is the first fractal we are going to investigate. Ask the students to think about how they would draw it. There is a lovely amination of an infinite Sierpinski Triangle here: http://fractalfoundation.org/resources/what-are-fractals/ Or here https://www.youtube.com/watch?v=TLxQOTJGt8c	

<table>
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<td>10min</td>
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Ask the students to draw the Sierpinski Triangle. There are various levels of challenge here.

You could give your students a blank sheet of triangular paper; they would have to work out how big to make the triangle themselves.
Or
In the workbook there is an outline of the large triangle they can use to get them started.
Or
For added support you can take them through slides 16-21, which show each iteration of the triangle.

At each stage students should think about the fraction that is not coloured/shaded, and fill in the table in their workbooks. The questions are on slide 22.

Encourage the students to look for a pattern in fractions. Can they find a general formula?
<table>
<tr><th>Iteration</th><th>Unshaded</th><th>Shaded</th></tr>
<tr><td>1</td><td>$\frac{3}{4}$</td><td>$\frac{1}{4}$</td></tr>
<tr><td>2</td><td>$\frac{9}{16}$</td><td>$\frac{7}{16}$</td></tr>
<tr><td>3</td><td>$\frac{27}{64}$</td><td>$\frac{37}{64}$</td></tr>
<tr><td>4</td><td>$\frac{81}{256}$</td><td>$\frac{175}{256}$</td></tr>
<tr><td>5</td><td>$\frac{243}{1024}$</td><td>$\frac{781}{1024}$</td></tr>
<tr><td>n</td><td>$\left(\frac{3}{4}\right)^n$</td><td>$1- \left(\frac{3}{4}\right)^n$</td></tr>
</table>
</td>
<td>PowerPoint.
Triangular paper.
Colouring pencils.
Workbook p2/3.</td>
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<td>25min</td>
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We are now going to investigate the Koch snowflake.

Take the students through slide 22-27.
How is this fractal made?
How does it match the definition of a fractal?

There is a zoom animation here:
▪ https://www.youtube.com/watch?v=PKbwrzkupaU

Ask the students to think about the perimeter at each iteration of the fractal.
Assume the length of the triangle is 1 to start with.

Can they work out the perimeter at each stage?
They can do this into their workbooks.
<table>
<tr><th>Stage</th><th>Perimeter</th></tr>
<tr><td>1</td><td>3</td></tr>
<tr><td>2</td><td>4</td></tr>
<tr><td>3</td><td>$\frac{16}{3}$</td></tr>
<tr><td>4</td><td>$\frac{64}{9}$</td></tr>
<tr><td>5</td><td>$\frac{256}{27}$</td></tr>
<tr><td>6</td><td>$\frac{1024}{81}$</td></tr>
</table>
</td>
<td>PowerPoint.
Workbook p4.</td>
</tr>
</table>

	Ask the student to discuss: What happens to the perimeter after n iterations? $3\left(\frac{4}{3}\right)^{n-1}.$ What happens to the perimeter after infinite iterations? Since the snowflake is the curve you get by doing this process infinitely often, the length of the snowflake curve is infinity. Is the area of the snowflake finite or infinite? The amazing aspect of the Koch snowflake is that the perimeter infinitely increases. However, the area is bounded by a circle, which would also bound the original triangle, so you have an infinite perimeter enclosing a finite area.	
40min	Dragon Curves. Dragon Curves are created by following these instructions. 1. Draw a straight line, say 2cm long, but the larger this line, the more stages you can draw. This is the start of your dragon. 2. Now draw two lines so that the original line forms the hypotenuse of an isosceles right-angled triangle and erase the original line. This is the first stage. (See the diagram below.) 3. For the second stage, replace each of the lines from the first stage with two new segments at right angles so that the lines from stage one form the hypotenuse of an isosceles right-angled triangle. 4. The new segments are placed to the left then to the right along the segments of the first stage. 5. Continue this construction, always alternating the new segments between left and right along the segments of the previous stage. This generates the `dragon curve'. Ask the students to draw the first few stages of the dragon curve. How does the dragon curve match the definition of a fractal? Students have space to do this in p5/6 of their workbooks. The solutions are shown on slides 36-40. There is a nice animation here. http://mathworld.wolfram.com/DragonCurve.html	PowerPoint. Workbooks p5/6
Extension	Extension: (for most groups this is unlikely to fit into the time allocated for the lesson) Ask the students: ▪ Can you generate Pascal's Triangle to line 15. ▪ Colour in the odd numbers. ▪ What do you notice? If you colour in the odd numbers of pascal's triangle, this begins to create the Sierpinski Triangle. This is shown on slide 44.	PowerPoint. Workbook p7
48min	Plenary Slides 45-47 suggest a couple of real life uses of fractals.	PowerPoint.