Josephus Flavius - Notes

This session is designed to last about 50 minutes. The timings are approximate and will vary from group to group. You may find it easier to change the ‘minutes elapsed’ to the actual times of your presentation. The presentation starts and ends with slides showing why studying maths beyond GCSE is useful. The slides are repeated; you only need to show them once. The accompanying workbook should be printed as a booklet and can be reduced to A5 (A4 folded).

Required Knowledge:

- This lesson builds on knowledge of place value to think about binary. However, no previous knowledge of binary is needed.
- Students should know that $10^0 = 1$.
- Students should be able to look at sequences and spot patterns.

Resources:

- Magic cards resource sheet.
- PowerPoint
- Workbook

Objectives of session:

- To understand how to work in Binary.
- To solve a mathematical problem, and come to a general form of the solution using binary.

Time	Activities/Questions/Points to make	Resources
10 mins	Magic Cards. (slide 6) Use the resource sheet to create sets of magic cards. **(don’t give out the workbook yet)** Give out one set of magic cards to a student, ask them to pick a number up to 31 and tell you on which card numbers their number appears. (E.g. number 17 is on card 0 and card 4) You can then amaze the students by reading their mind and telling them what number they choose. To do this use the card numbers as a power for 2 and add for each card, so 17 is $2^0 + 2^4 = 1+16 = 17$. Another example: 25 is on cards 4, 3, and 0. So, to work it out from the card numbers it’s $2^0 + 2^3+2^4= 1+8+16 = 25$. Give the students some time to have a play with these, can they work out how it works? After the students have had some time lead a discussion on how these cards work. Have the students noticed a link between the number in the top left hand corner and the card number? *(These are the values of the powers of 2, (1,2,4,8,16), with the card number as the index).* Have the students noticed a link between the numbers in the top left hand corners of cards in which their chosen number appears? (*They should notice these numbers add up to their chosen number*). Can you the teacher actually read minds, or is there a mathematical way of working out the number they chose? *Hopefully the students will be able to work out the method you used!*	Resource sheet on magic cards. You may want to make up enough sets of these so that students can play the game in groups.

15 mins	Place Value and Binary. (slides 7-12) Ask the student what 495 mean? *(4 hundreds, 9 tens, 5 units)* What does this mean in terms of powers of ten? *(4 lots of 10^2, 9 lots of 10^1, 5 lots of 10^0)* How do we describe 3287 using powers of 10? Our whole number system is based on powers of 10. Suppose that instead of powers of 10, we decided to base our number system on powers of 2. What would the columns be, instead of hundreds, tens, units? *(32s, 16s, 8s, 4s, 2s, 1s)* (2^5, 2^4, 2^3, 2^2, 2^1, 2^0) So how could we write 17? <table><tr><td>2^4</td><td>2^3</td><td>2^2</td><td>2^1</td><td>2^0</td></tr><tr><td>1</td><td>0</td><td>0</td><td>0</td><td>1</td></tr></table> 10001 How could we write 25? <table><tr><td>2^4</td><td>2^3</td><td>2^2</td><td>2^1</td><td>2^0</td></tr><tr><td>1</td><td>1</td><td>0</td><td>0</td><td>1</td></tr></table> 11001 You can now give out the workbooks, and ask the students to use the table on page 1 to write some numbers in binary. (slide 13)	PowerPoint Workbooks.

Extra tasks on binary (you may choose to move on at this point depending on time).

1. Discuss if these statements are true for the decimal system, binary system, or both. (slide 14)

Are these true?

1 + 1 = 10
(yes in binary)
10 + 1 = 11
(yes in decimal and binary)
10 + 11 = 101
(yes in binary)

2. There are also some binary sums for the students to try. They can either do this by converting into decimal, adding, and converting back again, or they can think about how column addition would work in binary. (slide 15/16/17)

Key Addition Results for Binary Numbers
- 1 + 0 = 1
- 1 + 1 = 10
- 1 + 1 + 1 = 11

Key Subtraction Results for Binary Numbers
- 1 – 0 = 1
- 10 – 1 = 1
- 11 – 1 = 10

Question	Answer
111+100	1011
101+110	1011
1111+111	10110
111-101	10
110-11	11
1100-101	111
1110+10111	100101
1110+1111	11101
11111+11101	111100

15 mins	Tell the story of Josephus Flavius (slide 18) Josephus Flavius was a famous Jewish historian of the first century at the time of the Second Temple destruction. During the Jewish-Roman war he got trapped in a cave with a group of 40 soldiers surrounded by romans. The legend has it that preferring suicide to capture, the Jews decided to form a circle and, proceeding around it, to kill every alternate remaining person until no one was left. Josephus, not keen to die, quickly found the safe spot in the circle and thus stayed alive. We are going to investigate how he managed to stay alive. Show the example for a group of 10 people. (slides 19-30) Ask the students to have a play, and use the circles in their workbook to work who would survive for various group sizes. Circles are given for 6,8,11,12. Students can then draw their own circles for other group sizes. Make sure the students number their circles! (slide 31) Ask them to put their data in a table. (slide 32) Results (slide 33)	Powerpoint Workbooks.

Number of people (n)	1	2	3	4	5	6	7	8	9
Who wins? J(n)	1	1	3	1	3	5	7	1	3

Number of people (n)	10	11	12	13	14	15	16	17	41?
Who wins? J(n)	5	7	9	11	13	15	1	3	19

10 min

Discussion of the results.

Discuss with the students what they notice in the results. Ask the students to describe the patterns.

When does the pattern repeat itself?
(at 2,4,8,16)
What is the significance of these numbers?
(these are the powers of two we looked at earlier)
Given what we did earlier can anyone suggest an alternative way to look at this problem?
(Hopefully someone will suggest binary, as the powers of 2 are significant in this pattern).

Ask the students to rewrite their tables using binary. (slide 34)

n	1 1	2 10	3 11	4 100	5 101	6 110	7 111	8 1000	9 1001
J(n)	1	01	11	001	011	101	111	0001	0011

n	10 1010	11 1011	12 1100	13 1101	14 1110	15 1111	16 10000	17 10001
J(n)	0101	0111	1001	1011	1101	1111	00001	00011

What is happening here? Can you spot how to get between the number n, and the winning space to stand J(n)
(the solution is you move the first digit of the number to the end, slides 35-39 show this)

What is actually happening when we are moving the first digit to the end?

Consider 495, changing to 954 in decimal. (slide 40/41)

First we do 495 subtract 400, to give us 95.
Then we are multiplying by 10, to give us 950.
Then we are adding 4 to give 954.

Powerpoint
Workbooks.

	So what is happening in binary? (Slide 42) Consider 1011. (11 in decimal) First subtract the highest power of two. (removing the one in the far left column, which is worth 8) Then multiply by two, to move everything left one place value column. Then we add 1. Summary: Number of people – (biggest power of 2 possible, 0,1, 2, 4, 8, etc) Then x 2 Then +1 When there were 41 people, where did Josephus stand? (slide 43) 41-32 = 9 9 x2 = 18 18+1 =19 Stand in place 19. Ask the students to check this works with their original results in their table. Do the students think that Josephus actually did this to save himself? The last slide is a mathematical joke. (slide 44) There are **10** types of people in the world, those who understand binary and those who don't.	
	Extension There are some lovely Binary Cross numbers available here: http://www.cleavebooks.co.uk/trol/trolwj.pdf	