the **Further Mathematics** network

Schools, Colleges, Universities and Local Authorities working together to support and promote Advanced and Further Mathematics.

*Why it means so much for mathematics…*
the **Further Mathematics** network

Supporting and Promoting
Advanced and Further Mathematics

“The increase in A level Further Mathematics in just three years is an astonishing achievement. It will make a huge difference both to the students going on to university and to the departments who will be teaching them.”

“The Further Mathematics Network can justifiably claim to have had a significant impact, not only on entries to the subject, Further Mathematics, but also on participation in mathematics more generally.”

**Professor Celia Hoyles, Department for Children, Schools & Families, Chief Adviser for Mathematics**

“Further Mathematics is vital to stretch and inspire mathematically able students and a very valuable qualification for all who wish to take maths-related subjects at university. The work of the Further Mathematics Network in ensuring that all students have access to study Further Mathematics means that more students will be able to make the most of their mathematical potential.”

**Professor Adrian Smith, Principal, Queen Mary College, University of London**

“It was only when I did Further Mathematics at school that I got a first glimpse of what mathematics was really about. The Further Mathematics Network has the potential to provide an exciting bridge between mathematicians in University at the cutting edge of their subject and the next generation of mathematicians coming through school.”

**Professor Marcus du Sautoy, University of Oxford**

“If we are to compete effectively in a global economy the country needs more graduates in Science, Technology, Engineering and Mathematics. These subjects are completely dependent on students being fluent and confident in mathematics. Students who have studied Further Mathematics before they go to university are very well-prepared to deal with these maths-related degrees. This year’s A level figures show that the work of the Further Mathematics Network in promoting AS/A level Mathematics and Further Mathematics, and giving all sixth formers the opportunity to study Further Mathematics, is already having a very positive effect.”

**Sir Peter Williams, Advisory Committee on Mathematics Education**

“The Further Mathematics Network is doing an excellent job. Engineering relies heavily upon mathematics so that, when students begin an engineering course with a Further Mathematics qualification, whether at A level or AS level, we find they are significantly better prepared to manage their studies.”

**Dr John Morton, Chief Executive of the Engineering and Technology Board**
the **Further Mathematics** network

Supporting and Promoting Advanced and Further Mathematics

Programme Leader - Charlie Stripp

The Further Mathematics Network is working to help all students reach their potential in mathematics.

To do this we are widening access to appropriate tuition and providing support for teachers to help them to teach AS/A level Mathematics and Further Mathematics with confidence.

The Network is also helping to promote mathematics, to make sure that every student is aware of the opportunities that studying AS/A level Mathematics and Further Mathematics makes available to them.

To achieve this, the Further Mathematics Network works in four main areas:

**Providing Further Mathematics Tuition**
The Network provides Further Mathematics tuition to students when their school or college is unable to offer it. Further Mathematics reinforces, extends and deepens the standard AS/A level Mathematics. It is a prestigious qualification, greatly valued by universities and employers.

**Revision Days**
The Network provides Revision Days for AS/A level Mathematics and Further Mathematics. These are often held at universities and students from several different local schools/colleges attend. Presentations and activities give a complete overview of a module, emphasising understanding and giving advice on exam technique. Teachers who attend these events also report that they find them useful.

**Enrichment for Key Stage 4 and Post-16**
The Network sets up and supports opportunities for students to explore and think about mathematical topics outside the standard school curriculum. This could be through mathematics enrichment events, often held at local universities, or by providing access to resources and advice.

**Support for Teachers**
The Network’s extensive online resources to support AS/A level Mathematics and Further Mathematics are available free to schools/colleges in England. The Network also supports professional development for teachers at Key Stage 4 and post-16. This can be as simple as email and telephone support for teachers new to a module, through to full courses covering pedagogy and mathematical content.
The Further Mathematics Network is a government-funded initiative. It is managed by Mathematics in Education and Industry (MEI), an independent educational charity. The Further Mathematics Network consists of 46 Further Mathematics Network Centres. Within its region, each Centre aims to ensure AS/A level Further Mathematics is available to all AS/A level Mathematics students, promote mathematics to all young people and help support the teaching of AS/A level Mathematics and Further Mathematics in all schools and colleges.

The Further Mathematics Network, combined with the change to the structure of AS/A level Further Mathematics, has already led to a significant increase in the number of students achieving Further Mathematics qualifications. Each of these students’ exposure to Further Mathematics has made them more aware of their own potential, the pleasures of an intellectual challenge and the extra opportunities available to them because of their mathematical ability. Schools and colleges report that involvement with their local Further Mathematics Network Centre has raised the profile of mathematics amongst their students. We need more students to study mathematics post-16 and we need more well-qualified students to choose maths-related subjects at university. The Further Mathematics Network is already starting to make this happen.

Go to www.fmnetwork.org.uk for details of your local Centre.
Providing Further Mathematics Tuition

The Further Mathematics Network enables all students in schools and colleges in England to study Further Mathematics by providing tuition when their school or college could not otherwise offer it.

All AS/A level Mathematics students who intend to study for a maths-related degree (Engineering, Sciences, Computing and Finance, as well as Mathematics itself) should have the opportunity to study AS/A level Further Mathematics.

- Access to AS/A level Further Mathematics is vital to challenge and inspire mathematically-able students, who are not sufficiently stretched by the standard A level Mathematics on its own.

- AS level Further Mathematics is also very valuable for those that aren’t the highest fliers (perhaps studied over 2 years or in year 13). It introduces important new topics that ease students’ transition to maths-related university courses and improves their standard A level Mathematics grades.

The Further Mathematics Network aims to give every student who could benefit from studying Further Mathematics the opportunity to do so.

Further Mathematics Network Centres use various models for teaching Further Mathematics to students studying in schools and colleges that do not offer the subject directly.

Under-pinning all of the models is a comprehensive website that helps to structure students’ learning. The web resources allow students to study effectively, even when contact time with a teacher/tutor is limited. The resources are designed for use alongside textbooks and include opportunities for frequent formative assessment.

Exactly how tutorial support is organised depends on local circumstances and can vary significantly between and within Centres. The Centres try to operate flexibly to meet local need. Classes may be organised in local venues where students meet regularly with a tutor, or tutors may visit students in their schools/colleges on a weekly or fortnightly basis. Live tutorials given via the internet are also being used very successfully. Students can access support by email and usually get together at least once a term for a ‘study day’, often at a local university, where they have an intensive day of tuition focused on particular topics or on revision for module exams.
What students and teachers say about tuition through the Further Mathematics Network

Our results demonstrate that this method of studying, with limited direct teacher contact time and a strong emphasis on independent learning, can be very effective. It also helps the students to prepare for university-style study:

Emma Rose, studied Further Mathematics with the Coventry and Warwickshire Further Mathematics Centre

“I found doing Further Maths at A level to be extremely beneficial when studying Physics at university. A degree in Physics requires you to have a strong knowledge and understanding of maths, and a Further Maths A level really helps to prepare you for this. In my first year, a two semester long module was spent covering topics that are taught in Further Maths. Those that were not familiar with the higher level of maths had to devote more of their time to learning it, and I noticed that quite a few really struggled. I feel this was because the teaching methods at A level are more interactive and personal, and I imagine it would have been a lot harder to get to grips with the harder maths modules had I been taught them at university. I would strongly recommend doing a Further Maths A level to anybody who plans to take Physics at university - not only because it teaches the mathematical concepts required, but because it is the best environment to learn them in.”

Anil Ghandi, studied Further Mathematics with the Black Country Further Mathematics Centre

“I’m currently in my second year studying Mathematics at the University of Oxford. I did the Further Maths course with the University of Wolverhampton and found the course extremely worthwhile; I would definitely recommend the course to any sixth form student who is likely to do a maths-related degree at university.

The way the course was structured, being given the material and sent to work on your own, with supporting tutorials, was a great way to prepare for university. I was also quite fortunate to have had an extra hour’s tutorial type session in school for any other problems I had that weren’t covered in the weekly session at the university.

Although most universities said that Further Maths wasn’t required to do a maths related degree I have found it a great advantage to have done the course, I feel that any extra maths, to the single A level is a great help. A lot of the material covered in the further maths course has now become second nature to me (no matter how hard I found it at the time).”
Varunjay Ahluwalia, studied Further Mathematics with the Greater Manchester Further Mathematics Centre

“This course has been really beneficial to me, not least because it gave me the opportunity to learn Further Maths which was unfortunately unavailable at my college and also because it enabled me to achieve a higher standard in A level Maths. It is different from other methods because it depends on self-study rather than taking notes in class. In this way, I think it is much more effective than traditional methods. It can motivate students not only towards their chosen subject but also towards learning as a whole.

The course itself has been thoughtfully structured, starting with easy work and building a sound knowledge base, developing essential techniques like problem solving and analytical thinking. It offers other worthwhile opportunities such as interaction with other able students and making new friends. This means education and socialising going hand in hand, making it a different experience altogether.

Another noticeable difference in this course is that students can do their lessons and assignments in their own time, much like university.

Although it is based on self-study, help is always at hand. The website has been particularly helpful as students can access it at any time and of course my tutor has always been available to help and assist. I would like to show my appreciation for the guidance given by the teachers in their subject area.

Personally I think the course is a step forward in taking education out to people and giving them the choice, showing them what is on offer! I would recommend this course to anyone who, like me, is planning to take a degree in engineering. I feel much more confident now in my chosen career.”

Julian Merson, Head of Mathematics, Longsands College

“Longsands College has benefited immensely from working with Cambridgeshire Further Mathematics Centre. Our problems were that we lacked sufficient interest in Further Mathematics in order to make it a viable A level on the curriculum. That meant that we lost a few good students to other schools/colleges which did offer it. It also meant that other students who might have been in a position to benefit from Further Mathematics, never thought about it as a prospective field of study. We don’t lack graduate expertise within the department, although our teachers are now not experienced at teaching Further Mathematics.

The Cambridgeshire Further Mathematics Centre offered a flexible and cost neutral approach which appealed both to our students, and to the school. The first year of running this scheme has led to a marked increase in interest from other students in taking Further Mathematics in some form. We are now able to offer students at GCSE, who are in a position to take early entry GCSE, a real extension curriculum right through to Further Mathematics in Years 12/13. Eventually, I would like to take the course in house, but appreciate, nonetheless, the flexibility the Cambridgeshire Further Mathematics Centre offers in being able to support any student who wishes to pursue further study in Mathematics.”
David Paine, studied Further Mathematics with the Berkshire Further Mathematics Centre

“I took Further Maths because I really enjoyed Maths at GCSE and I found I had a real aptitude for it. It’s been a really rewarding experience and it’s given me a real insight into higher level maths. The Further Maths Network has been really useful in helping me to take Further Maths, as they have provided resources that I found very helpful during homework and revision.

Although we have a single lesson per fortnight, the quality of teaching and the atmosphere in lessons is very good and I feel confident coming up to my exams in a week’s time.”

Jean Helsby, Mathematics Teacher, Lymm High School

“Thanks to the Further Maths Network, we have finally managed to get Further Maths back onto the timetable at Lymm. Numbers had dwindled so that classes were no longer viable, but in October 2002 it was arranged for one of our gifted mathematicians to be tutored via video-conferencing (during the pilot phase of the Further Mathematics Network). We were then able to register with the Manchester centre in 2003. Since then numbers have gradually increased, we have moved from AS to A level and had sufficient numbers to return to timetabled lessons in-house for year 12 in September 2007. Without the help and support of the Further Mathematics Network this would not have been possible; the students have valued both the tuition and study/revision sessions available to them.”

Katie Liew, studied Further Mathematics with the London West Further Mathematics Centre

“The Further Maths Network was very supportive throughout the course of my study, providing out of class help, email correspondence when needed and organising some helpful module specific revision days.

Like most who have taken Further Maths through the Network, I found it highly rewarding and worthwhile as it is an impressive and valuable qualification to hold.”

Robert Day, studied Further Mathematics with the Berkshire Further Mathematics Centre

“I joined my school’s Further Maths class by chance, a few weeks into Year 12; I saw Further Maths mentioned on a university prospectus and asked my Statistics teacher about it, who put me in touch with the after-school class. It’s been an excellent experience, introducing me to some more interesting mathematics, and learning more advanced calculus and trigonometry has also made my regular Maths stronger.”
A key aim of the Further Mathematics Network is to enrich students’ experience of mathematics wherever possible. There is particular focus on students at Key Stage 4 and post-16. Formats used to provide such enrichment include:

- Workshops/hands-on sessions
- Quizzes and competitions
- A level Mathematics & Further Mathematics and University Mathematics taster sessions
- Student conferences
- Residential courses
- Video conferencing
From event feedback forms:

“I wanted to let you know that the students who attended yesterday’s Further Maths Day found it to be an enjoyable and challenging experience. I haven’t been to such a day before and was a little apprehensive about it, but the whole day was interesting, well presented and engaging at a level the students can relate to. I would certainly be interested in bringing students to an event like this again.”

“The Further Mathematics Network has helped to re-introduce an interest in the subject and helped to show that it is accessible to many of our students.”

Peter Taylor, Head of Mathematics, Caistor Grammar School

“Thanks for a brilliant day on Tuesday. Our students found it a very enriching experience and the reports back indicate that they had a great time. The three lectures that I saw were excellent - all challenging, thought provoking and very informative. Speaking to my second in department, after arriving back in Caistor we both found the day very rewarding for both us and the pupils. I certainly feel that as a result of the lectures that I will be able to teach more effectively in certain areas. Well done to you and your staff. You said you wanted it to be a fun day and you certainly succeeded in this.”

Diane Conley, Head of Mathematics, Wintringham School

“Thank you for giving my students the opportunity to experience the Further Maths Enrichment Day. My students all enjoyed the day and in their feedback to me they all showed how they had gained something special from the experience. They particularly talked about the application of the mathematics and felt the best parts were when they were able to get involved.

For my part I was amazed at the length of the journey in mathematics that was put into such a short time and in such an understandable way - iteration to complex numbers and Pythagoras to polyhedra.

The emphasis on proof and why it’s so important was also very applicable as this is an area which they are just coming to terms with. You put a lot of hard work into organising the day and I want you to know how much it was appreciated.”

Go to www.fmnetwork.org.uk for details of enrichment opportunities organised by the Further Mathematics Network.
Revision Days

Revision Days are a great opportunity for students to spend a day on intensive examination preparation.

A Revision Day usually focuses on the content of a specific unit, e.g. Core 1, Further Pure 2. Sometimes Revision Days are for one awarding body’s specification only but often students are split into groups so that the day can cover more than one specification.

The majority of Revision Days are at universities, organised through the local Further Mathematics Network Centre. This has many benefits; school and college students value greatly opportunities to experience learning in a ‘different, new and inspirational’ environment that a university can bring. Revision Days are also an excellent professional development opportunity for teachers. They provide a very time-efficient means for teachers to get an overview of a unit, from basic content through to addressing tricky aspects of examination questions.

---

**NM 19th June 2006**

**Question 7**

The following values of the function $f(x)$ are known.

<table>
<thead>
<tr>
<th>$x$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>3</td>
<td>5</td>
<td>7</td>
<td>9</td>
</tr>
</tbody>
</table>

It is required to estimate $D = f(2.5)$ and $\int_{1}^{4} f(x)\,dx$.

(a) Use the forward difference method to estimate $D$.

(b) Use the Lagrange interpolation to obtain the best possible estimate of $f(2.5)$.

(c) Construct a table to which the values at part (b) have the most plausibility.

---

**The main ideas are**

**Table 1**

<table>
<thead>
<tr>
<th>Energy</th>
<th>Work done</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$</td>
<td>$W$</td>
</tr>
<tr>
<td>$E_1$</td>
<td>$W_1$</td>
</tr>
<tr>
<td>$E_2$</td>
<td>$W_2$</td>
</tr>
<tr>
<td>$E_3$</td>
<td>$W_3$</td>
</tr>
<tr>
<td>$E_4$</td>
<td>$W_4$</td>
</tr>
</tbody>
</table>

---

**Gravitational potential energy (GPE)**

The work done by a constant force $F$ in lifting an object from $x$ to $x'$ is given by $W = F \Delta x$. Therefore, the gravitational potential energy of a body of mass $m$ at a distance $r$ from the center of the Earth is $U = -\frac{GmM}{r}$, where $G$ is the gravitational constant, $M$ is the mass of the Earth, and $r$ is the radius of the Earth. The work done by a constant force $F$ in lifting an object from $x$ to $x'$ is given by $W = F \Delta x$. Therefore, the gravitational potential energy of a body of mass $m$ at a distance $r$ from the center of the Earth is $U = -\frac{GmM}{r}$.

---

**Graphs of energy levels**

The graphs show the energy levels of a system as a function of position. The energy levels are given by $E_n = n^2 \hbar^2 / 2m$, where $n$ is a positive integer and $\hbar$ is the reduced Planck constant. The energy levels are equally spaced and the spacing is determined by the quantum number $n$. The graphs show the energy levels as a function of position. The energy levels are given by $E_n = n^2 \hbar^2 / 2m$, where $n$ is a positive integer and $\hbar$ is the reduced Planck constant. The energy levels are equally spaced and the spacing is determined by the quantum number $n$.
From event feedback forms:

“All our students found the FP1 Revision Day useful. Would attend any relevant Revision Days in the future.”

“It cemented my understanding.”

“A very concise and helpful way to revise.”

“Much clearer now, networks and linear programming were particularly good. Excellent day!”

“Lots of tips and points to look out for which will be really useful when teaching this unit.”

John Finlayson, who attended a Revision Day at the Berkshire Further Mathematics Centre

“Just want to say thank you very much for organising the FP1 Revision Day in June. I felt so confident after it I was hardly nervous at all going into my exam - feeling more eager to get stuck into it. I came out with an A overall, getting 89/100 in the exam. Please send my thanks to the presenters.”

Lisa Biddlecombe, who attended Revision Days at the Coventry and Warwickshire Further Mathematics Centre

“You mix with people from other schools - it is a great way to meet people with similar interests and make new friends.”

James Thompson, who attended Revision Days at the Greater Manchester Further Mathematics Centre

“The Revision Days were invaluable and something that a normal college or sixth form does not provide. I also found these days gave me an experience of what it would be like in university and I had to be self-motivated to succeed at the course just as I do now at Liverpool University.”

Go to www.fmnetwork.org.uk for details of Revision Days organised by the Further Mathematics Network.
Support for Teachers

All School/College Mathematics Departments (including those that already offer Further Mathematics) can access the following support through their local Further Mathematics Network Centre.

- Free access to expert advice on:
  - any issues associated with teaching and learning AS/A level Mathematics and Further Mathematics;
  - progression to university courses in Sciences, Technology, Engineering and Mathematics subjects (STEM);
  - supporting mathematically-able students at Key Stage 3 and 4 (this means that it is useful for 11-16 schools to register with the Further Mathematics Network).

- Information about any professional development activities organised by their local Further Mathematics Network Centre and across the national Further Mathematics Network.

- Free access to extensive online resources (by registering) for AS/A level Mathematics and Further Mathematics units. There are separate resources tailored to match the AQA, Edexcel, OCR and OCR(MEI) specifications.
From event feedback forms:

“Enjoyed looking at the online resources and activities - can we have more of the same?”

“Sessions gave a good overview of many topics, ideas for teaching, pointing out students’ difficulties and typical exam questions. Also liked interactive ideas for complex numbers and matrices.”

**Gillian Ward** attended the ‘Introduction to AS and A level Mathematics’ course at the University of Bath organised by the Wiltshire Further Mathematics Centre.

“I really enjoyed the course, which I attended with two colleagues. We love the resources you showed us, in particular the visual proofs, excel AGs/GPs.”

**Jill Jaxon** attended a Decision Mathematics session provided live over the internet by the Further Mathematics Network.

“That was really helpful. I’d been bothered by the two-step simplex and I feel I can explain what’s happening much better now. It was great fun and very worthwhile.”

**Simon Clay** took the Teaching Further Mathematics course.

“I have found the whole course excellent, interesting and helpful. It has been well organised and all the tutors and administrators have been exceptionally good.”

**Sue Coleman** took the Teaching Further Mathematics course.

“I am much impressed by the quality of the web resources, textbooks and lecturers. The lecturers have been most encouraging and supportive. I would strongly recommend this course to other maths teachers.”

Go to [www.fmnetwork.org.uk](http://www.fmnetwork.org.uk) for details of support for teachers organised through the Further Mathematics Network.
Charlie Stripp
Programme Leader
The Further Mathematics Network
MEI, Oak House, 9 Epsom Centre
White Horse Business Park
Trowbridge, Wiltshire
BA14 0XG
Tel: 01392 435 604
Fax: 01392 459 474
Email: charliestripp@fmnetwork.org.uk

Janice Richards
National Administrator
The Further Mathematics Network
MEI, Oak House, 9 Epsom Centre
White Horse Business Park
Trowbridge, Wiltshire
BA14 0XG
Tel: 01225 774 777
Fax: 01225 775 755
Email: janicerichards@fmnetwork.org.uk