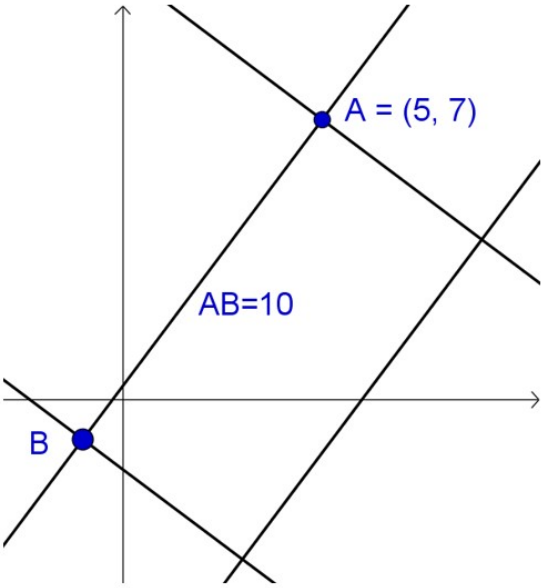
Integrating Problem Solving into A Level Mathematics Lessons: Developing Students' Questioning Skills 1

Aim	To develop the students' ability to ask useful questions in order for them to be able to identify • The information that they can see immediately • The information they think they will need • The mathematical skills that they will need to use So that they are more confident when encountering mathematical problems
Method	Asking students to come up with suitable questions to help solve a problem is often met with silence as students don not always have the ability to focus on what they need. This activity gives the students a reason and permission to ask questions. By letting the students know that some information missing from a problem so it is currently unsolvable gives them something on which to focus.
Activity	The students are presented with a mathematical problem from which some important information is missing. The teacher makes it clear that the task is to develop the students' problem-solving skills as well as ultimately solving the problem. The teacher informs the students that there is some vital information missing and that they will have to ask questions to find that information. The teacher should also point out that some obvious questions such as "what are the coordinates of the other points?" will not be answered. The students should be given some time (up to two minutes) to consider the problem without writing anything down. They should then spend a little more time (again up to two minutes) writing down as many questions as they can that would give key information for solving the problem. Students should then compare their questions with those of one or two other students. They should try to say why they think their question will reveal some important information. The students are then invited to ask the teacher their questions and the teacher should answer these questions being careful not to give the solution to the problem away The teacher should encourage the students to consider the answers to each question and how that may help them to solve the problem. At each stage the teacher can give prompts to help the students think, in particular encouraging them to combine pieces of information The students attempt to solve the problem.
The example used in the video is shown on the next page.	



Find the equations of the straight lines in this diagram.

AS/A Level

Understand and use the equation of a straight line, including the forms $y - y_1 = m(x - x_1)$ and $ax + by + c = 0$; gradient conditions for two straight lines to be parallel or perpendicular.

Possible student questions:

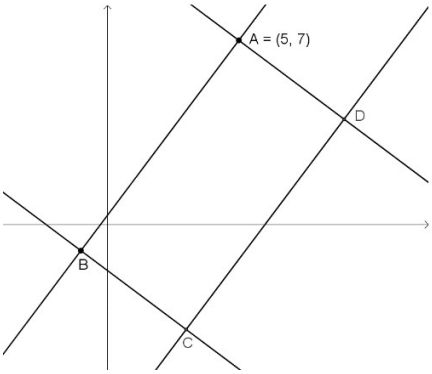
- Does the line through AB pass through $(0,0)$ – is the diagram just badly drawn?
- Are those lines perpendicular? Are those lines parallel?
- Do we know the coordinates of any of the other points?
- Do we know the lengths of any of the other line segments?
- Are the scales on the x axis and the y axis the same?
- Are the x and y coordinates of each point integer values?
- Is there only one solution?

This approach can be applied to any number of problems and is remarkably easy to set up. To develop your own additional resources, remove key information from any exam or specimen paper problem solving questions.

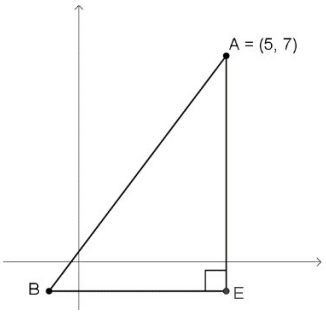
For additional ideas and transcripts of lessons in which these techniques have been applied, see the MEI's Mathematical Problem Solving: A Guide for teachers at mei.org.uk/problem-solving-guide.

Solution to the problem

Labelling the points of intersection A, B, C and D as shown:



For $AB = 10$ the right-angled triangle AEB (shown below) must be a Pythagorean triple with 10 as the hypotenuse since all of the points are at integer values. There is only one triple that has this property, a $\{6,8,10\}$ triangle.



From the fact that the line passes above the origin (i.e. the gradient of $AB > 1$) it can be seen that $AE > BE$ so from the Pythagorean triple $AE = 8$ and $BE = 6$ giving the point B at $(-1, -1)$.

Note: were the lengths swapped i.e. $AE = 6$ and $BE = 8$, the point B would be at $(-3,1)$ which it clearly isn't as it is below the x axis.

The gradient of $AB = \frac{8}{6} = \frac{4}{3}$. The gradient of $BC = -\frac{3}{4}$.

For integer coordinates we now have a $\{3,4,5\}$ triangle or one that is a multiple of $\{3,4,5\}$. Since $BC < AB$ (which the students should have established in their questioning), only a $\{3,4,5\}$ is possible.

This places point C at $(3, -4)$ and, since AD is parallel to BC and AB is parallel to CD, point D at $(9,4)$.

The equations of the lines can now be found

The line through AB: gradient $= \frac{4}{3}$, through $(-1, -1)$ $\quad y + 1 = \frac{4}{3}(x + 1)$

This simplifies to $4x - 3y + 1 = 0$.

The line through CD: gradient $= \frac{4}{3}$, through $(3, -4)$ $\quad y + 4 = \frac{4}{3}(x - 3)$

This simplifies to $4x - 3y - 24 = 0$.

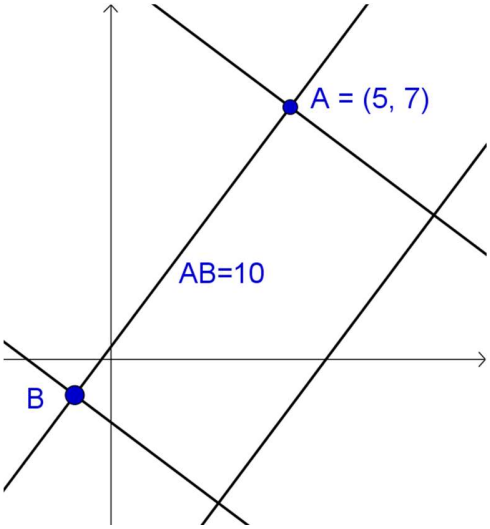
The line through BC: gradient $= -\frac{3}{4}$, through $(-1, -1)$ $\quad y + 1 = -\frac{3}{4}(x + 1)$

This simplifies to $3x + 4y + 7 = 0$.

The line through AD: gradient $= -\frac{3}{4}$, through $(5,7)$ $\quad y - 7 = -\frac{3}{4}(x - 5)$

This simplifies to $3x + 4y - 43 = 0$.

Problem solving in A level Mathematics



Find the equations of the straight lines in this diagram.