



Qualifications and
Curriculum Authority

Offering further mathematics as part of the A level curriculum

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Contents

Introduction	3
Why offer further mathematics at all?.....	5
For students	5
For the institution.....	6
For the staff.....	7
Summary.....	7
How do we decide on entry requirements?	8
How can we fit further mathematics into an already crowded A level provision?	10
Example 1 – School offering further mathematics in the second year of a double mathematics course	10
Example 2 – School offering further mathematics alongside A level mathematics.....	11
Example 3 – Further education college offering further mathematics as a fully timetabled A level option.....	11
Example 4 – Shared provision with other local institutions.....	12
Example 5 – Shared provision with a local further mathematics centre	13
Example 6 – Tuition entirely through a local further mathematics centre	13
Flexibility	14
Suggested models	15
What approaches can we use in the teaching and learning of further mathematics?.....	17
What resources are there to support teaching and learning?.....	20
Textbooks.....	20
Using technology to enhance teaching and learning.....	20
Interactive animations	21
Online resources available through the Further Mathematics Network.....	21
Useful websites	21
<i>Improving learning in mathematics</i>	22
Past papers.....	22
How can we support our staff in teaching further mathematics?	24
Subject knowledge	24
Subject pedagogy	24
How do we attract students to the course?	26
How do we support students during their course?	27
Successes and challenges.....	28
Appendix 1	30
Data	30
Appendix 2	31
Useful contacts.....	31

Introduction

Further mathematics enjoys a unique status in the A level curriculum. It is not free-standing as it cannot be certificated before the equivalent qualification in AS or A level mathematics has been achieved. Further mathematics, usually perceived as a minority subject, was lost for some schools and colleges in the changes that resulted from the introduction of Curriculum 2000 with its emphasis on breadth. Students who previously had been studying three subjects at A level were now expected to study four or even five in their first year. Finding time to take on further mathematics in addition to this became problematic for both institutions and their learners. The uptake of mathematics – already seen as one of the hardest AS subjects – declined, as did the study of further mathematics. Changes to the A level mathematics specifications in 2002 have reversed that decline somewhat but many of the issues for the provision of further mathematics have remained:

- **viability** – for centres with a small number of A level mathematics students, forming a viable group of further mathematicians may not be economically viable
- **staffing** – finding mathematics teachers with the confidence and expertise to take on the additional challenge of teaching further mathematics
- **profile** – awareness of potential students (and their parents) of the existence of the course and the benefits of studying it.

In 2004 the Qualifications and Curriculum Authority (QCA) issued a questionnaire to schools and colleges to collect data on the provision of A level mathematics and further mathematics (including AS qualifications in these two subjects). Approximately 200 questionnaires were returned by centres and telephone interviews were conducted with 45 of these to obtain more information about the provision of further mathematics in the organisations within which they worked. Four further mathematics centre¹ managers were also interviewed by telephone (although they were not included in the initial survey). In addition, brief face-to-face interviews were conducted with 22 further mathematics students from three classes.

This report is intended to help schools and colleges that do not at present offer further mathematics as part of their AS/A level provision but are considering offering it in the future. It is also intended to help centres already offering further mathematics that would like to know the strategies used by other centres when dealing with issues such as small class sizes and the professional development of staff. It may also help those centres considering widening

¹ The further mathematics centres are part of the Further Mathematics Network funded by the Department for Education and Skills (now the Department for Children, Schools and Families) with a remit to widen participation in further mathematics.

participation in further mathematics. Examples of practice from individual centres are used to illustrate the suggestions.

Why offer further mathematics at all?

All centres were very positive about their decision to offer and maintain further mathematics provision at their school or college. They talked of benefits to their students, to their institutions and to their mathematics teachers.

For students

A current further mathematics student in his second year of study, when asked why he was studying further mathematics, replied, 'If you really like maths then you want to do *more* maths!' This sort of interest, motivation and excitement was a common reason that teachers cited for maintaining their provision.

All the centres interviewed that had contact with former students at university who had gone on to study mathematics or a mathematics-related subject such as engineering, science or economics invariably reported that their ex-students had benefited from studying topics not included in A level mathematics:

Absolutely positive about it! They are so glad they have studied further mathematics. One lad who went on to study engineering came back and said that it was so useful for him – it didn't make him top of the class but just to be familiar with things like complex numbers gave him real confidence in his first year... He saw other people struggle who hadn't done further mathematics.

Head of mathematics, 11–18 school

A current further mathematics student was asked what advice he would give to someone who was thinking about whether they should do further mathematics:

It depends what you want to do after. I'm going to do engineering and they use those j numbers all the time so I'm glad I've done them.

A2 further mathematics student

Some centres expressed concern about the response of some higher education admissions officers to applicants having further mathematics on their application form. Of particular concern was the response from medical schools that did not include further mathematics qualifications in their offers. In 2006 the Further Mathematics Network released a statement intended for admissions officers of medical schools that concluded, 'it would be advantageous to all students, whether or not they proceed to read medicine, if all medical schools were now to accept AS further mathematics'² and Imperial College Medical School has recently agreed to accept AS further mathematics as part of their offer.

² The full text of the statement can be found www.mei.org.uk/files/pdf/FM_and_medical_schools.pdf.

Most centres reported that the reaction from higher education institutions has recently become much more positive:

At the beginning of this year, one of my A2 students who wasn't doing further mathematics begged me to allow her to join the AS [further mathematics] class because she realised it would help her UCAS application.

I think we may be in an upward spiral now. A few years ago many of the universities didn't seem interested [in whether our students had further mathematics or not] so fewer did it and so universities seemed even less interested. Now our students are getting really positive responses so more are taking up the subject and the universities begin to expect it more.

Head of mathematics, 11–18 school

For a number of centres the additional challenge that further mathematics offers to mathematics A level students was key to maintaining it as part of their A level offer:

For our really good students the A level isn't enough! They are capable of much more and should have this opportunity.

Head of mathematics, sixth-form college

For other centres the benefits of taking further mathematics extended into A level mathematics:

For our students who do further maths and only have a B [at GCSE] it has really helped with their [mathematics] A level too. Because they are exposed to so much maths every week it gives them familiarity. They do better, I think, because of it.

A level mathematics coordinator, further education college

Current students seem to agree:

I did OK in my first year [doing the AS mathematics course] but this further maths course has really helped my understanding. We do lots of algebra in this class that looks scary but when we go through it, it isn't, so I don't get freaked out when I see stuff in my C4 module.

Student taking further mathematics AS in her second year

We've done a couple of things in this class that I thought were really hard but then when the same topic came up in the normal class I went, 'Yeah! This is so easy!'

Student studying AS mathematics and further mathematics concurrently

For the institution

Many centres have continued to offer further mathematics, even when it has been difficult to timetable due to low numbers. They reasoned it had tangible benefits to their school or college:

We wanted to be seen as a serious academic institution. If we don't offer further maths and other local sixth forms and colleges do we are going to lose out on enrolling the best mathematicians.

A teacher from a sixth-form centre that made a decision to start offering further mathematics three years ago added:

The year we put further maths on the timetable as a proper subject we saw a big increase in the number of potential A level maths students applying to the sixth form.

A level mathematics coordinator, sixth-form college

For the staff

For those interviewed, the opportunity to be able to teach further mathematics was seen to be very positive. Teachers talked about their enjoyment of being with a group of students who were highly motivated and one teacher commented:

After all I wouldn't be a maths teacher if I wasn't interested in maths. My further maths group share that interest... Very satisfying.

Further mathematics teacher, 11–18 school

Another teacher saw it as a challenge:

When you have people in front of you in the class and it could be they are better mathematicians than you are, it keeps you on your toes!

Further mathematics teacher, 11–18 school

Not one school or college interviewed for the survey regretted the decision to offer further mathematics but several heads of department had at some time had to defend its position as part of their institution's A level offer, particularly when only small numbers were recruited.

Summary

Further mathematics:

- benefits students who go on to study mathematics or a mathematics-related subject at university
- helps to recruit students to the institution
- offers an additional challenge to A level mathematics students
- can help students with their A level mathematics course
- is valued by universities
- can motivate staff.

How do we decide on entry requirements?

Before changes in 2004, the specifications for further mathematics required prior knowledge and understanding of much of the content of the mathematics AS and A2 modules, which meant that it was very difficult to teach any of the further mathematics modules before the prerequisite modules in A level mathematics had been completed. Since 2004 the revised specifications have enabled the awarding bodies to offer AS further mathematics in a form that makes it feasible to teach alongside AS mathematics. AQA and OCR (including the Mathematics in Education and Industry (MEI) option) already have specifications in place which make this possible. Edexcel will introduce new further mathematics specifications ready for first teaching in September 2008 and examinations in 2009.

The changes to the specifications are also intended to make the study of AS further mathematics more accessible to those students who may not have previously been considered as candidates, and some centres have changed their entry requirements in the light of this.

From those interviewed for this report, 37 out of 45 centres had an entry requirement for studying further mathematics as a grade A or A* at GCSE mathematics, and a third of these only offered places to those students with a grade A*. The other eight centres said they did not have any particular entry requirements other than those for studying A level mathematics, but reported that it was rare for students to be in their classes without at least a grade A at GCSE. All four of the further mathematics centres interviewed had no particular entry requirements and accepted students on their home institution's recommendation.

Those centres that had a grade A or A* at GCSE as their entry criterion for joining a further mathematics class cited as their reason the students' ability to cope with the course, either because of the content itself or because of the reduced class contact time:

We have taken students with grade B [at GCSE] in the past but they struggled with the content and dropped out. We are more careful now about who we allow on the course.

Head of department, state sixth-form college

It's the pace. We have less time so the pupils have to be really good to start with.

Head of department, state school

In addition, the centres using Edexcel as their awarding body said that the prerequisite to starting the further pure 1 module was completion of the modules C1, C2, C3 and C4.³ It was considered

³ This is due to change from September 2008.

unlikely that anyone with less than a grade A at GCSE would be able to cope with that level of work.

Those centres that had a more flexible approach to entry cited motivation as the main factor in determining whether a student would be successful on the course. Such centres have become less restrictive in their entry requirements since the changes to the specifications were introduced. One head of department at a state sixth-form college outlined their entry requirements:

If they have a C [at GCSE] then they are not advised to do the further maths course although we have had success with two students who came into the class with a C... They need to have a little bit of algebra... We never put a B student off.

Head of department, sixth-form college

Another teacher from a further education college with less restrictive entry criteria commented:

We could cherry-pick our further maths students – only take A*s for example – whizz through the course, lose a few here and there who can't keep up and end up with all grade As all round. But the changes [to the specifications in 2004] have given us a great opportunity to open up maths to youngsters we wouldn't have thought were further maths students in the past...The further maths AS is not much harder than the normal one and if you have the flexibility to let year 13 students pick it up – their algebra has really improved by then – it can really add value to their UCAS application.

A level mathematics coordinator, further education college

How can we fit further mathematics into an already crowded A level provision?

In many cases further mathematics is treated differently to other A level options in schools and colleges. It is often seen as an 'extra' to a student's A level package. Low numbers mean that for financial reasons a number of schools and colleges allocate less time to teaching further mathematics than they allocate to A level mathematics – in some cases this could mean as little as one hour face-to-face contact each week formally timetabled for further mathematics students. In cases like this the teacher often supplements the allocated time by providing workshops or after-school sessions in their free time.

Further mathematics is offered in schools and colleges in various ways ranging from being offered to students as a standard option on the timetable to an informal offer that is negotiated with senior management each year depending on the numbers who choose to take it up. For those centres with small numbers of students or without staff to teach it, they can be supported by their local further mathematics centre.

Below are some examples of how schools and colleges have managed their provision.

Example 1 – School offering further mathematics in the second year of a double mathematics course

The school uses the Edexcel specification that has, as a prerequisite for the further pure 1 module, knowledge of the content of the pure modules from A level mathematics. The school offers an option of double mathematics to candidates joining the sixth form. In the first year all six modules of A level mathematics are studied in one and a half times the normal AS time. In the second year either three or six additional modules are studied for an AS or A level further mathematics qualification. The time allowed for the second year is equivalent to the time given to study the three modules of other A2 courses regardless of whether they will study three or six modules to complete either the AS further mathematics or the full A level.

The school expects the students to complete the full A level further mathematics at the end of the second year but will be flexible with individuals who may not need the full A level for entry to higher education or who have struggled to achieve the top grades in A level mathematics. Entry to this course is by interview in addition to having attained a grade A or A* at GCSE – the school sees this class as suitable for 'natural mathematicians only' owing to the reduced contact time available for it.

A concern about the low numbers of students opting to take further mathematics led the school to introduce this model three years ago. The teacher commented:

We sell this as a 'get an A level in one year' course! We identify potential students lower down the school and encourage them to take the course... Because they are doing so much maths together they really gel as a group and few drop out at the end of year 1.

Head of mathematics, 11–18 school

Occasionally, students from the AS mathematics class join the group in the second year but they are expected to have worked through C3 and C4 independently over the summer break to prepare them for the class. These students typically attain an AS further mathematics qualification at the end of the course.

Example 2 – School offering further mathematics alongside A level mathematics

This school uses the AQA specifications. Students are not in a separate class for their A level mathematics lessons but can join any of the classes that fit in with their other A level choices. Their further mathematics is taught in a weekly two-and-a-half hour session on one afternoon a week. At the end of year 1 these students will have been examined in three modules for AS mathematics and three modules for AS further mathematics. Year 2 follows the same pattern.

The school likes the flexibility of this model as it also allows students to opt into further mathematics AS at the end of their first year of study if they wish. Again, the fact that the content is delivered over a reduced contact time and that the students who take the course are doing so in addition to the four subjects that all sixth formers have to take means that the school generally asks for A or A* at GCSE. However, in the past they have allowed students with grade B at GCSE to join the class but they must have completed AS mathematics with a top grade and need to pick up an additional qualification in their second year.

Example 3 – Further education college offering further mathematics as a fully timetabled A level option

This college uses the OCR (MEI) specifications and runs AS further mathematics and A2 further mathematics as standard options on the A level timetable. It allocates the courses the same amount of time as AS and A2 mathematics. Students can choose to take the AS further mathematics in their first or second year of study provided it does not clash with their other options.

This is the third year that the college has run the further mathematics courses this way. In previous years further mathematics was offered on an ad hoc basis to students who showed an interest and was delivered in a workshop style during lunchtime or at the end of the day. The college has also experimented with offering further mathematics as part of an intensive mathematics class similar to the model used in Example 1 above. This option was abandoned after the first cohort as, for this college, it led to a high drop-out rate and also excluded those students whose mathematical maturity was not at a level where they could even attempt the full A level in year 1.

The college adopted their current model when the specifications for further mathematics were changed so that studying the AS did not require candidates to have completed A level mathematics first. The head of department states that although they would prefer candidates to have at least an A grade in their GCSE mathematics they do not exclude anyone who is studying A level mathematics from joining the class if they wish. Last year the further mathematics AS class had 15 students with A or A* at GCSE, two students with grade B and one student with a grade C.

The college is keen to encourage students with interest and motivation to study as much mathematics as they can:

At the end of the day if they are keen and willing to do it, it seems silly not to give them a chance. We're not expecting them to get a grade A but we *do* expect them to pass.

Head of mathematics, general further education college

Example 4 – Shared provision with other local institutions

An 11–18 girls' school works with a nearby boys' school to provide further mathematics tuition to their students. The schools are following the OCR specifications. Students from both schools get together one afternoon per week for a two-hour class. In one week the venue and tuition is provided by the girls' school and in the next week the venue and tuition are provided by the boys' school. Neither school has a large enough cohort to justify running further mathematics independently of each other. The teacher at the girls' school commented:

It works well. I do the pure modules and H. teaches the applied. Our schools are very close so it is not a problem for the students. The only problem is I only see the boys once a fortnight but they know they can come in and see me at lunchtimes and after school and they have my email address... We would prefer to be able to run our own course but this is better than having no provision at all.

Further mathematics teacher, 11–18 school

Schools and colleges that offer further mathematics to their students do not have to use the same awarding body for both A level mathematics and further mathematics. This may be helpful when considering shared provision with other institutions.

Example 5 – Shared provision with a local further mathematics centre

A consortium of four schools works with their local further mathematics centre to provide tuition to 35 students. The further mathematics centre manager helps to coordinate the course and provides a tutor for one of the AS modules. The schools provide tutors for the other two modules.

This arrangement allows the schools involved to pool their resources and expertise and offer further mathematics to their A level students despite small numbers in each individual school. In addition, the flexibility of tutor support from the further mathematics centre can plug any gaps in the provision caused by timetabling difficulties.

These schools use the OCR (MEI) specifications for their further mathematics provision.

Example 6 – Tuition entirely through a local further mathematics centre

For some schools, providing further mathematics for their students is not feasible because of low numbers or lack of available teachers. In cases like these many have opted to enlist the support of their local further mathematics centre⁴ for all their students who want to study the A level or AS.

A typical arrangement would be that once a potential further mathematics student has been identified the further mathematics centre manager will arrange tuition for that student in a convenient local venue. The tuition is provided either on a one-to-one basis or as part of a class that the further mathematics centre has organised for any schools and colleges in the local area. Face-to-face tuition may be as little as one hour per week but this is supplemented by access to online teaching and learning resources, revision and study days, and email access to their further mathematics tutor.

Using a further mathematics centre to provide tuition is financially neutral to schools and colleges that use it: the further mathematics centre charges a fee for each module taught which is less than the amount the home institution claims back for each student through the normal funding

⁴ Local further mathematics centres are part of the national Further Mathematics Network (see Appendix 2 for contact details).

mechanisms. (The fee is set at this reduced level in recognition of any administrative costs such as examination entry fees that the home institution may incur.)

As one head of department commented:

It doesn't cost us anything overall. If we didn't offer further maths we couldn't claim any money anyway... This way we can give our two very good mathematicians the chance to study further maths without it affecting our budget. We couldn't do that if we tried to provide tuition ourselves – apart from the fact that I don't have the available staff... They want to study maths at university.

Head of mathematics, 11–18 school

A further mathematics centre manager explained the role of the centre:

The ideal situation for me would be for all schools and colleges to offer further mathematics in-house as part of their A level offer but I know that's unrealistic at present and for some it may never be possible... What's happening in one school is that last year I did all the tutoring for the course but this year they are teaching one module themselves. The intention is that at some point in the future they will do it all themselves... The teachers have built up confidence in the unfamiliar modules by attending the revision and study days I do for the students. As interest in further maths goes up I'm hoping more and more of the schools will take it on themselves.

Further mathematics centre manager

Flexibility

For many students the decision to take further mathematics as part of their A level package when they first enter the sixth form is clear cut in that they have enjoyed success at GCSE, mathematics is their favourite subject and they want to study mathematics when they go on to higher education. But for others the decision is not so easy. It may be that:

- the students have no clear idea of what they want to do post-18
- the students' original plans for study at university are not mathematics-related but they later change their mind
- the students enjoy mathematics but would rather study a broader range of A levels
- at the end of their first year the students are very successful in the AS mathematics course and think they can capitalise on their mathematical maturity by studying more mathematics rather than continuing with subjects in which they are less successful.

Below are three strategies that centres have used to grant access to further mathematics to such students:

- fully timetabled further mathematics classes where the AS modules are delivered in year 1 and the A2 modules delivered in year 2 allows students to pick up AS further mathematics in their second year of A level study (assuming it does not clash with their other options)

- supported self-study with face-to-face contact scheduled outside the normal timetable allows students to pick up the additional modules they need for a further mathematics qualification
- use of the local further mathematics centre to provide tuition and support for modules that students would not have access to due to timetabling difficulties.

Some teachers commented on the difference between AS further mathematics and A2 further mathematics and suggested that for some students just studying for the AS qualification would be appropriate. Two current further mathematics students had some advice:

The second year is tough, so I'd say if you're quite good at maths and interested do the AS, but if you're very good at maths and very interested do the whole thing.

A2 further mathematics student

What I'd say is: further maths [AS] is not harder but different. If you're happy with your A level maths work you'll be happy with this. Don't not do it just because you think it's harder.

AS further mathematics student

Schools and colleges that have a flexible approach to their provision may be in a better position to cater for these two types of student.

Suggested models

If your centre can in all likelihood expect only a small number of candidates, consider the following:

- forming a consortium with other local schools and colleges in your area to create a viable group size
- using your local further mathematics centre to provide tuition for all or part of the course
- using a supported self-study model where a reduced contact time is supplemented by the use of online resources such as those offered through the Further Mathematics Network.

If your centre does anticipate a large enough cohort of A level mathematicians to have a fully timetabled class for further mathematics, the model you can choose will depend on your entry requirements and the flexibility you would like for joining the class. Examples 1 and 3 above illustrate how two institutions manage their provision.

One centre, which recently decided to offer further mathematics, put it on their timetable in the same way as the other A levels without an expectation of having an economically viable class size in the first year:

We made a decision, supported by senior management, that we were going to run further maths on the timetable, allocate it equal time to the other courses and allow anyone who had

chosen maths to pick it up. Of course we interviewed them carefully – looked at their other options, tried to make sure they were motivated. We ran it as a 'loss-leader' in the first year – only had nine students – but the college started to get the reputation as being the place to come to if you wanted to do further mathematics. Our numbers have gone up and we now have a healthy 18 students in the AS and 12 in the A2.

Head of mathematics, further education college

What approaches can we use in the teaching and learning of further mathematics?

Many mathematics departments find themselves in the position of having to teach further mathematics with far less time than is given to the A level mathematics course and many feel this has an effect on how best to deliver the course.

Most respondents who were interviewed described the teaching and learning model they used for delivery of the further mathematics curriculum as traditional: the teacher introduces a topic, works through some examples with the students and then sets exercises for homework. The most common reason given for using this transmission model was the amount of content that needed to be covered in a short amount of time:

We have a lot of content to cover and are trying to do it on a reduced timetable... I try to make it interesting and ask lots of questions but my eye is constantly on the clock.

Head of mathematics, 11–18 girls' school

Some teachers who worked in this way said they were aware that progress through the further mathematics specification could be seen 'as a grind' and commented on the ways in which they had tried to add value to the student experience by, for example, introducing historical contexts to new topics, trying to make their initial exposition a more shared experience by asking questions of the students and encouraging them to ask their own questions. One teacher described his approach as 'flexible':

I go by what they [the students] want. Sometimes they want me to just get on and explain a topic step by step, other times they want to stop, have a discussion, talk about how it fits in with other work they've done. I'm really open to suggestions from them and because we have such a good relationship it seems to work.

Further mathematics centre manager

Some teachers had thought about introducing more active approaches such as group work, learner-to-learner discussion and open questioning to lessons but had not pursued this mainly because of perceived time constraints. In addition, there was a general feeling that further mathematics students do not need to work in this way:

Because our students have excellent GCSE grades and are highly motivated – real further mathematicians really – I can tell them something and they get it straight away... Those that don't [get it] come and see me at lunch or after school and I go over it again.

Further mathematics teacher, 11–18 boys' school

A number of centres talked of selecting their potential students carefully to ensure they had a cohort that could work comfortably this way and indeed all schools and colleges that had an entry requirement of a grade A* at GCSE and used mainly a transmission model for teaching reported

that their students invariably were awarded an A or (more rarely) a B grade in their further mathematics A level or AS.

Teachers who had recently made changes to the way they were approaching the delivery of their A level mathematics course, through initiatives such as the Department for Education and Skills (DfES) (now the Department for Children, Schools and Families) Standards Unit *Improving learning in mathematics*,⁵ had all, at the very least, considered using the same approaches with their further mathematics classes:

We were seeing the benefits to our A level students of working in a more active way and have tried to incorporate some of the approaches into further maths.

A level mathematics coordinator, further education college

Some teachers believed that even the most talented students would benefit from using more active approaches to teaching and learning:

There is a time and a place for developing mathematical rigour and it is an important part of preparing students for studying mathematics at university, but I have tried to introduce a balance into my further maths teaching. The proportion of class time I spend on things like matching activities, sorting activities – tasks that are a little out of the ordinary – is less than I would spend on [mathematics] A level but it is important to have a mixture... I have seen grade A students that can't think outside the box... They've not developed their thinking skills so when they meet something a little out of the ordinary they are at a loss as to what to do.

Further mathematics teacher, sixth-form college

Schools and colleges that had more flexible entry requirements were more likely to incorporate activity-based learning into their teaching:

We have a wider range of ability in the AS class now, so we have been thinking about how to engage those students without reducing the challenge to the really clever ones. It's hard but we've started to develop 'thinking' resources that are accessible yet challenge all our students... We also use mini-whiteboards a lot and ask open questions like 'show me an example of...' and things like that.

A level mathematics coordinator, further education college

One teacher, who was conscious of the time pressures due to reduced class contact time, had thought carefully about how to use resources that encourage discussion and collaboration:

Homework doesn't have to be exercises! We know practice is essential in developing mathematical fluency but that doesn't mean every single homework has to be churning out answers to exercises. Often I want them to do something a bit different and, ideally, I would do it in class with them if we had more time.

⁵ See the section 'What resources are there to support teaching and learning' for more information about this initiative.

I set them an activity to do for homework – either on their own or in a group. Then at the beginning of the next lesson, rather than just going over it and checking that they have got the correct matchings, add a short extra dimension/twist as the starter for the lesson. For example, if their homework was to investigate graphs of the form

$$y = \frac{A}{(x-b)(x-c)}$$

either by using computers or matching cards... Then the challenge [at the start of the next lesson] would be to sketch the graph of something of the form

$$y = \frac{A}{(x-b)(x-c)(x-d)}$$

They would do this in pairs... This would create some discussion, would check their homework via discussion and get them thinking about the understanding of what they've done. And best of all would not take too long!

Alternatively, if they have had to do an activity on their own that needs checking let the starter be a group of four comparing answers and coming to a consensus through discussion about what the correct version is.

This idea could be extended to any sort of problem-solving activity – even something as simple as finding the errors in a piece of work or giving out a solution (to a problem) that has a series of questions on it – like a sort of mathematical comprehension. Then a lot of thinking has been done (before they come to the lesson) but discussion is generated to resolve the contentious/challenging parts of the activity.

Further mathematics teacher, sixth-form college

What resources are there to support teaching and learning?

Textbooks

The teachers in the survey talked about using textbooks as an additional resource to the work going on in the classroom, to be used for reference and for providing exercises for homework. They generally preferred to have a textbook that had been written specifically for the specification they were following as they felt it gave confidence to their learners to know that 'everything you need to know is contained within these pages'.

One teacher described using the textbook in the following way:

Before the start of each topic I sometimes give them this homework: I ask the students to work in pairs, read through the explanation in the textbook and try out a couple of examples and then prepare a presentation for the rest of the class on the topic. No one knows which pair I will ask to do the presentation on the day so they all have to do it – I surprised one pair by asking them to do another presentation the week after they had just done one because I didn't want the class to think they wouldn't get asked again. That one wasn't too good! Sometimes the presentations are a bit of a muddle but, because everyone has had to do some preparation, the class together will sort it out.

Further mathematics teacher, further education college

Using technology to enhance teaching and learning

The majority of centres interviewed reported that they used technology of some sort to enhance the teaching and learning of further mathematics. Some had access to classrooms with interactive whiteboards that allowed them to use software packages (such as those for graph drawing and geometry) interactively with the whole class.

Teachers used these packages to enhance their whole-class teaching:

The functionality of these packages has increased enormously in the last few years. If I am doing something like inequalities with [the class] I can prepare the functions in advance so I can reveal them step by step on the interactive whiteboard. I get them to predict what is going to happen to the graph at different points along the x -axis and then I can instantly reveal it on the board... Having the functions pre-prepared and revealed dynamically means I can concentrate on the questions I'm going to ask rather than on my own graph drawing... And for things like 3D vectors – being able to rotate the point of view, to see vector equations of lines, intersections, and planes has made a big difference to some of my students. It has brought vector geometry out of their heads so we can talk about it.

Further mathematics teacher, 11–18 school

Teachers also encouraged their learners to use the software for investigative tasks:

I set a homework last week which was: 'Here are some functions. Find me some polynomials that can model each function between -1 and 1 making sure your absolute error is always less than 1 per cent'. They worked with the software in the library and the next lesson we discussed their results.

Further mathematics teacher, 11–18 school

Interactive animations

Several teachers mentioned the use of Java applets in their teaching of the mechanics units. These are generally available on the web for free use (see Useful websites, below).

One teacher, who is running a further mathematics class on reduced contact time, uses the applets in the following way:

I email my students a PowerPoint document where I have pasted in links to the websites where they can find the applets. I ask them to play around with the applets for a while, changing numbers or forces and so forth and then I have a series of questions about what is going on. At the start of the next lesson we have a discussion and they compare answers and so forth.

Further mathematics centre manager

Online resources available through the Further Mathematics

Network

Schools and colleges that register with their local further mathematics centre have free access to resources to support the teaching and learning of further mathematics, including materials for the applied A level modules. These extensive resources support all four of the English awarding bodies' further mathematics specifications.

Free registration with your local further mathematics centre is available at

www.fmnetwork.org.uk/register.php.

Useful websites

The following are web addresses of online sites that the centres interviewed mentioned as being useful to them in delivering further mathematics:

Content	Web address
Mathematics activities, puzzles and a forum for asking for help with mathematics questions	www.nrich.maths.org
Collection of interactive resources covering the further mathematics specification	www.mathsnet.net
Rich starting points for investigations at A level	www.risps.net
Resources for teaching further mathematics topics	www.plymouth.ac.uk/math
Online resources covering the further mathematics specifications for all awarding bodies	www.mei.org.uk
Details of enrichment activities for further mathematics students	www.fmnetwork.org.uk
Java applets for mechanics	www.walter-fendt.de/download/ph14dl.htm

Improving learning in mathematics

In 2005 the DfES Standards Unit published *Improving learning in mathematics* which is a free resource⁶ containing over 50 session plans and a programme of professional development for mathematics teachers working in the post-16 sector. Although the session plans are mainly aimed at learners studying for GCSE and A level mathematics, a number of those interviewed reported that they were adapting the ideas and approaches exemplified in the resource for their further mathematics classes.

Past papers

All of the centres used past papers as a resource for teaching and learning. One teacher who is a member of her regional subject learning coach network⁷ recalled one of the meetings last year:

⁶ To order a copy of the resource, go to www.ncetm.org.uk.

⁷ For information about subject learning coaches go to www.subjectlearningcoach.net.

We had a session on active revision where we thought about how we could use past papers in different ways so that instead of just handing out past papers and then going over the answers we could use them in different ways... I tried some of them out and it was brilliant!

Subject learning coach, further education college

Suggested activities using past papers include the following:

- Working in pairs, hand out the answers and ask students to reconstruct the questions.
- Give each pair a question from a past paper and ask them to create the mark scheme then compare their mark scheme with the official one. This can help identify aspects of the questions that the examiners feel are important and award marks for.
- Underline key words and phrases in the questions and ask pairs to explain them – these can be general instruction words like 'hence' or specific mathematical words like 'determinant'. This can help identify areas for further revision as well as reinforcing the meaning of instruction words.
- Hand out a solution to an exam question with the steps in the wrong order and ask pairs to rearrange the steps to construct the answer. For an additional challenge you can miss out some steps and ask the learners to write them in themselves.

How can we support our staff in teaching further mathematics?

Subject knowledge

Teaching further mathematics requires a greater level of subject knowledge than is required to teach A level mathematics. There are topics in the further pure modules such as complex numbers or matrices that don't appear at all in A level mathematics, as well as familiar topics such as calculus which are delivered at a higher level. In addition, teachers may also need to extend their knowledge of mechanics, statistics or decision mathematics to be able to teach enough applied modules to complete the course.

For some centres finding staff with appropriate subject knowledge was problematic and heads of department tended to rely on their experienced staff to teach further mathematics. One head of department commented:

We've been lucky really. We have two very experienced teachers who have taught further maths for years... One of them is close to retirement now so we will have to think about what to do to replace him.

Head of mathematics, 11–18 school

Others described strategies they had used to ensure they or their teachers had appropriate subject knowledge:

- Encouraging staff to attend subject knowledge courses.
- Mentoring of less experienced staff by other members of the mathematics department. For example, one further education college operates a buddy system where new teachers are allocated a named member of staff who has experience of teaching the further mathematics modules. The new teacher plans lessons and works through example questions with his or her buddy to ensure the concepts are well understood by the new teacher.
- Releasing teachers to attend sessions run by the awarding bodies.

Subject pedagogy

For most schools and colleges interviewed, the issue of professional development for their further mathematics teachers was centred around subject knowledge – what to teach rather than how to teach. Interestingly, few had ever observed colleagues teaching further mathematics except in rare cases as part of management-led appraisal. A common reason given for this was that there was so little further mathematics going on in the school or college that there were few opportunities to engage in peer observation. Some larger mathematics departments, however,

had in place a comprehensive peer observation programme for mathematics teaching that included further mathematics.

Colleges with subject learning coaches reported that they discussed ways to teach in their network meetings where they met other further mathematics teachers. Some of the centres that were working towards widening participation in further mathematics to students without A or A* at GCSE had joined networks where they could discuss introducing a wider variety of methods and activities than they would normally consider using if they had stricter entry requirements.

A further mathematics centre manager is organising a working group for teachers in her local area to get together to plan sessions and evaluate their effectiveness.

All centres with more than one further mathematics teacher shared resources and lesson ideas for teaching further mathematics among themselves and one school, which had a regular slot in its department meeting to discuss teaching and learning, ensured that further mathematics was included as part of that.

For teachers in smaller departments, finding the time or opportunity to discuss the pedagogy of teaching further mathematics was more problematic, but it was important to some to meet with other further mathematics teachers and engage in discussion. Some suggestions for how to do this are below:

- join your local Further Mathematics Network (www.fmnetwork.org.uk)
- join a local network of mathematics teachers (if you don't know of any, you can contact the regional coordinator at the National Centre for Excellence in the Teaching of Mathematics (www.ncetm.org.uk), who will have information about networks in your local area)
- complete a subject knowledge course, as many include elements of pedagogy
- include opportunities to discuss teaching and learning on meeting agendas
- join a professional subject association (see Appendix 2)
- join an online discussion community such as those at www.ncetm.org.uk.

How do we attract students to the course?

Most centres interviewed reported a decrease in the numbers taking up further mathematics following the introduction of Curriculum 2000, although numbers have started to recover since then. Many have put in place strategies for attracting more students and have seen a subsequent increase in candidates. The strategies used are designed to increase awareness of further mathematics and generate interest and enthusiasm. Below are examples of the strategies used by the schools and colleges interviewed for this report:

- Create a display at parents' evenings that highlights career opportunities for those who go on to study mathematics or a mathematics-related subject at university and how opting to take further mathematics can be of benefit – include quotes from students and ex-students.
- Use current further mathematics students at options days to talk about their experience of studying further mathematics.
- Use further mathematics students who have gone on to university to talk about the benefits of studying further mathematics.
- Organise enrichment days/sessions for years 10 and 11 (your local further mathematics centre has information about local events).
- Offer the OCR additional mathematics Free-Standing Mathematics Qualification (FSMQ)⁸ to year 11 students. This can be done in a number of ways:
 - One school has a class that has already completed the specification for GCSE mathematics and is expected to achieve grades A or A*. The school provides an additional challenge to these pupils by teaching the content of the FSMQ in normal mathematics lessons.
 - Another school that has identified prospective further mathematicians in more than one of their GCSE classes puts on an extra class off the timetable and encourages pupils to attend this.
 - One further education college works with local feeder schools to identify potential candidates for A level mathematics and further mathematics, and offers a weekly FSMQ additional mathematics class at the college.
 - A further mathematics centre runs a weekly FSMQ additional mathematics class after school that pupils from local schools can attend.

⁸ The additional mathematics FSMQ is administered by OCR. For further details, see www.ocr.org.uk/qualifications/FreeStandingmathematicsQualification-Additionalmathematics.html.

How do we support students during their course?

Given that so many schools and colleges teach further mathematics in fewer hours than allocated to the A level mathematics course, support for learners outside formal classes may be crucial to learner success. Most of the support offered is of an informal nature; teachers give up lunchtimes and after-school time to see students individually. In addition to this, some schools and colleges offer further support and suggestions for their students:

- One school encourages study groups for their further mathematics students. The teacher sets tasks for homework that require learner collaboration such as preparing presentations, making posters or doing practical mechanics experiments using simple equipment.
- A large further education college has a well-used mathematics workshop that runs daily at lunchtime or after college and, although open to any student studying mathematics at any level, is staffed by a further mathematics teacher on one session a week.
- One school encourages its further mathematics students to use the NRICH⁹ website, which has discussion boards where users can post mathematics questions and discuss their solution.
- One school has invested in access to the MEI online resources (which cover further mathematics modules from all the awarding bodies).
- Other schools and colleges take their classes to revision and study days provided by their local university or further mathematics centre.

⁹ www.nrich.maths.org.

Successes and challenges

All those interviewed were asked what they considered to be successful about their provision of further mathematics and foremost among the successes cited were student achievement and enjoyment. In addition to this, many felt that the present climate for studying further mathematics was more positive than it had been at the introduction of Curriculum 2000. Increased interest from universities, they felt, was fuelling increased interest by potential students and that, coupled with the changes to the specifications in 2004, making the AS further mathematics more accessible, had supported their individual strategies for increasing participation in the subject.

Here are some of the reported successes:

- Effectively employing strategies to increase the number of students studying further mathematics.
- Making changes to teaching and learning to cater for a wider range of ability in the further mathematics AS class and developing resources to support these changes.
- The mathematics department working together to support each other in introducing further mathematics to the curriculum.
- Successfully promoting an interest in further mathematics to pupils lower down the school.
- Enjoying the support of the senior management team in maintaining the provision of further mathematics.
- Getting further mathematics put on the sixth-form timetable rather than running it on an ad hoc basis from year to year.

Heads of department also commented on the importance of the commitment and motivation of their teaching staff in maintaining provision and enriching the experience of their A level mathematics students.

There were, of course, challenges which still faced some of those interviewed:

- maintaining provision in the face of low numbers
- developing resources for teaching and learning
- overcoming the perception of further mathematics as being too hard.

But a final comment from one further mathematics teacher is indicative of the commitment of those interviewed to continue to provide further mathematics for their students:

Sometimes I think that a teacher's best resource is another teacher. So if you think further maths is a good thing – and I believe there are lots of reasons to think so – then you find ways of offering it despite the difficulties you might encounter. Talk to teachers in other colleges and schools – see how they do it.

Head of mathematics, sixth-form college

Appendix 1

Data

Institutions interviewed by telephone were self-selected from a sample of approximately 200 schools and colleges that responded to the QCA questionnaire which led to the interim report, *Evaluation of participation in GCE mathematics*, in 2005. Further details can be found at www.qca.org.uk/qca_10451.aspx.

Forty-five interviews were conducted consisting of:

- twenty state 11–18 schools
- nine further education colleges (including sixth-form colleges)
- fourteen independent/fee-paying schools
- two combined maintained/fee-paying schools.

Of these, 23 were using the Edexcel specifications, eight were using the AQA specifications, six were using the OCR specifications and eight were using the OCR (MEI) specifications. Four further mathematics centre managers were also interviewed by telephone. In addition, 22 current further mathematics students were interviewed during site visits to three classes. Interviews took place between January and April 2007.

Appendix 2

Useful contacts

Awarding bodies

AQA	www.aqa.org.uk
CCEA	www.ccea.org.uk
Edexcel	www.edexcel.org.uk
OCR	www.ocr.org.uk
WJEC	www.wjec.co.uk

Professional subject associations

The Association of Teachers of Mathematics	www.atm.org.uk
The Institute of Mathematics and its Applications (IMA)	www.ima.org.uk
The Mathematical Association	www.m-a.org.uk
National Association of Numeracy and Mathematics in Colleges (NANAMIC)	www.nanamic.org.uk
Royal Statistical Society	www.rss.org.uk

Other mathematics organisations

Advisory Committee on Mathematics Education (ACME)	www.royalsoc.ac.uk/acme
The Further Mathematics Network	www.fmnetwork.org.uk
Mathematics in Education and Industry (MEI)	www.mei.org.uk
National Centre for Excellence in Teaching in Mathematics (NCETM)	www.ncetm.org.uk