Problem Sheet 3 – Quadratics and Cubics Problems

Question 1	**Find the value of a such that the turning point of the parabola $y = x^2 - 2ax + 1$ is closest to the origin.**	**Source: UKMT**
Question 2	**Show that, if x is a real number then $2x^2 + 6x + 9$ is always positive.**	
Question 3	**Find the value of k such that the quadratic function of x** $$k(x + 2)^2 - (x - 1)(x - 2)$$ **is equal to zero for only one value of x.**	
Question 4	**Find all real solutions of the equation** $(x^2 - 7x + 11)^{(x^2 - 11x + 30)} = 1$	**Source: NRICH**
Question 5	**Find the smallest value of the function $f(x) = 2x^3 - 9x^2 + 12x + 3$ in the range $0 \leq x \leq 2$.**	
Question 6	**In this question b and c are real numbers.** **(a) By considering the graph of $y = x^2 + bx + c$ show that if $c < 0$ then the equation $x^2 + bx + c = 0$ has two distinct real roots. Is the converse statement also true?.** **(b) Determine the three conditions for the equation $x^2 + bx + c = 0$ to have distinct positive roots.**	

Question 7	**Let $f(x) = x^3 - 3px + 1$ where p is real.** **(a) Explain why the equation f(x) = 0 has at least one real root.** **(b) Using differentiation or otherwise, show that if $p \leq 0$, then the equation f(x) = 0 has one and only one real root.** **(c) If $p > 0$, find the range of values of p for each of the following cases:** **(i) the equation f(x) = 0 has exactly one real root,** **(ii) the equation f(x) = 0 has exactly two distinct real roots,** **(iii) the equation f(x) = 0 has three distinct real roots.**	